TEXTILE MATHEMATICS

PART I

BY

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PREFACE

The recent Education Act, in its relation to instruction in Continuation Schools and Classes, makes it desirable, if not absolutely necessary, that textbooks of an elementary nature should appear, such books dealing with the Science subjects most closely correlated to each large industry. Some knowledge of Elementary Science is essential, not only to the students of Continuation Schools and Classes, but to practically all who wish to prosecute intelligently the study of the technology of any particular branch of industry.

There are very few science textbooks which refer particularly to the textile industry, and it is hoped that this book—the first of two parts on Textile Mathematics—as well as other books which are to be published shortly, will supply, in part at least, this want in textile literature.

The general principles involved in all preliminary books on Mathematics are identical, although these principles may be explained in different ways. The present book, while embodying these common principles, differs from the usual books on the subject in that the principles are explained so as to appeal to textile students, while practically all the examples and exercises bear directly on the problems of textile work

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TEXTILE MATHEMATICS—I

CHAPTER I

INTRODUCTORY

Mathematics is the science of quantities, not merely of quantities in themselves, but also of the exact and definite relation between simple quantities and sets of quantities.

In textile work of all kinds, the solution of practical problems demands exact and definite relations between certain sets of quantities. To take a simple example, one speaks, say, of a No. 8 yarn; the significance of this number varies greatly according to the fibre of which the yarn is composed, and according to the district in which it is spun and woven. If the yarn is made of cotton fibres, No. 8, or simply 8^s, will mean that in 1 lb. of the yarn there should be 8 hanks, and each hank should measure 840 yd. in length. If the yarn is linen or flax, 8 leas of 300 yd. each should weigh 1 lb., while if the yarn be jute, 14,400 yd. should weigh 8 lb.

In each case the figure 8 expresses a definite relation between the length and the weight of a certain quantity of yarn, and it is usually possible to compare,

with little trouble, two or more yarns of different numbers. The number 8 actually indicates in the first two cases the relation between the lengths of yarn contained in a fixed weight, while in the case of jute, the same number shows the relation between a certain weight and a fixed length. All yarn tables are founded on one or other of these two bases; if it were practicable to number yarns according to their diameters, such a basis would possess obvious advantages over the others when dealing with questions affecting cloth structure.

Practically every person has at least an elementary knowledge of mathematics, although the knowledge may not be recognized by that name. In many instances the mathematical principle involved is applied only to particular cases, and no notion of its general application is entertained. It is the purpose of the following pages to show the mathematical principles underlying many kinds of textile calculations, and to show the application of general mathematical ideas to what are apparently intrinsically different problems.

CHAPTER II

SIGNS AND DEFINITIONS

In the manipulation of quantities there are, in reality, two elementary ideas involved—that of addition and that of subtraction; for multiplication may be regarded as a quick method of continued addition, while division may be regarded as a process of continued subtraction. In every case all quantities may be manipulated to obtain the desired results by one or

more of these four operations, viz. addition, subtraction, multiplication, and division.

To save time and trouble these operations are denoted by signs, as under:

The sign

$+$	known as	**plus**	stands for	**addition.**
$-$	,,	**minus**	,,	**subtraction.**
$\times$	,,	**multiplied by**	,,	**multiplication.**
$\div$	,,	**divided by**	,,	**division.**

Again:

The sign $=$ known as **equal to** stands for **equality.**
,, $\doteqdot$ is used for **approximately equal to.**
,, $\sim$,, to imply **difference between.**
,, ∞ means **infinity.**
,, $:$,, **is to.**

The undermentioned numbers and signs,

$$6 + 4 = 10,$$

are read, 6 added to 4 equals 10,

or, 6 plus 4 equals 10.

This is a "numerical equation", although a very simple one, and implies that the terms on the left-hand side of the sign $=$ are of the same value as the single term on the right of the same sign. In a somewhat similar manner we may introduce other simple numerical equations as under:

$$10 - 6 = 4,$$

and read, 10 minus 6 equals 4.

Again,

$$6 \times 4 = 24$$

means that 6 multiplied by 4 is equivalent to 24; while

$$24 \div 4 = 6$$

indicates that 24 divided by 4 is equivalent to 6.

It is important to note that however many numbers appear on each side of the equality sign (=), those on one side are exactly of the same value as the simple number or group of numbers on the other side; this statement is true for all the different kinds of signs attended by numbers or the like which are connected by the sign (=) of equality.

Many simple numerical equations need all the first five signs which appear in the above four examples. Thus

$$(6 + 4) - (3 \times 2) + \{(18 \div 9) \times 4\} = 12$$

means that the so-called "expression" on the left of the mark = would, when simplified, have the value 12.

On the left-hand side of the above equation are three pairs of numbers, 6 and 4, 3 and 2, and 18 and 9, enclosed in what are known as **brackets** ( ). The last pair, 18 and 9, and also their brackets and the number 4, are enclosed in a larger bracket { }, while a still larger group of bracketed sets may appear within brackets marked [ ]. It will be seen that it is necessary to have differently-marked brackets in order to avoid mistakes.

In the above equation the brackets, and indeed all the signs used, have exactly the same meaning as in ordinary arithmetic; arithmetic is simply a branch of the parent subject of mathematics. The brackets, in particular, are used to group together quantities which have to be treated as single simple quantities. Thus,

the above equation means that 6 is to be added to 4, giving 10; that 3 is to be multiplied by 2, giving 6; that 18 is to be divided by 9, giving 2, and this 2 multiplied by 4, giving 8; and further, that the product of 3 and 2, or 6, is to be subtracted from 10, giving 4, and, finally, 8 is to be added to this 4, resulting in 12, the answer or single value on the right. All these operations may be set down briefly in the following manner:

$$
\begin{array}{ccccccc}
(6 + 4) & - & (3 \times 2) & + & \{(18 \div 9) \times 4\} & = & 12 \\
10 & - & 6 & + & (2 \times 4) & = & 12 \\
10 & - & 6 & + & 8 & = & 12 \\
\multicolumn{3}{c}{\text{or } 4} & + & 8 & = & 12.
\end{array}
$$

The foregoing example of a numerical equation deals essentially with a particular case; in many instances general results are required, and these general results can only be obtained by using symbols instead of numbers. The most convenient symbols are the letters of the alphabet, although any kind of marks could be made to serve the purpose. When a group or groups of letters appear on the left side of the equality mark (=), and another set appears on the right side of the same mark, the arrangement is said to be an "algebraical equation".

An algebraical equation of the same form as the above numerical equation may appear as follows:

$$(a + b) - (c \times d) + \{(e \div f) \times b\} = x.$$

The bracket containing e and f may be removed first, and the equation simplified:

$$(a + b) - (c \times d) + \left\{\frac{e}{f} \times b\right\} = x.$$

Then all the brackets may be removed, and the equation stated as under:

$$a + b - cd + \frac{eb}{f} = x.$$

Note that cd means $c \times d$, the product of c and d; and that $\frac{e}{f}$ means $e \div f$, the quotient of e and f. The simplified form,

$$a + b - cd + \frac{eb}{f} = x,$$

is at present unsolved, and cannot be solved unless values are given to all the letters but one. Suppose that $a = 6$, $b = 4$, $c = 3$, $d = 2$, $e = 18$, and $f = 9$. Then we can find the value of x by replacing the various letters by their numerical values. Thus, we change

$$a + b - cd + \frac{eb}{f} = x$$

$$\text{to } 6 + 4 - (3 \times 2) + \frac{18 \times 4}{9} = x,$$

$$\text{or } 6 + 4 - 6 + 8 = x,$$

$$\text{whence } 12 = x$$

$$\text{or } x = 12.$$

The principal point to notice at this stage is the fundamental difference between the two examples. The arithmetical equation is a particular case, and the answer must always be 12, because the equation deals only with definite and concrete quantities. On the other hand, the algebraical equation is a general case, because x may be of different values, according as we change the values of the remaining letters, and the equation is correct irrespective of whatever concrete values may be assigned to the symbols. It is not

difficult to see that if, in the above example, $a = 5$ instead of 6, the value of x must be 11.

Exercises, with answers, on p. 107.

CHAPTER III

REMOVAL OF BRACKETS

The use of brackets is attended with one difficulty, that of their removal when an expression or an equation requires to be simplified. The following instructions should be carefully noted:—

1. When the sign immediately preceding the first bracket of a pair is positive (+), the brackets may be removed and the signs unchanged without altering the value of the expression. That is to say,

$$x + (y - z) = x + y - z.$$

This process may be proved as follows:—Let the lengths of the straight lines AB, BC, and CD or DC

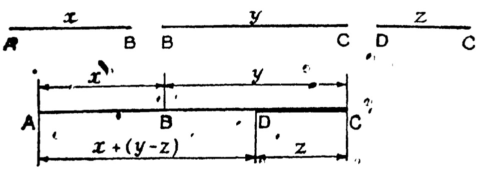

Fig. 1

(fig. 1), represent to any scale the values of x, y, and z respectively. Lines AB and BC from left to right may be considered positive, while CD, being from right to left, may be considered negative. To the length AB join the length BC, and cut off from the

latter the length CD. The length now remaining is clearly ABD. The result is evidently AD, and AD is therefore equal to $x + y - z$. Now AB $= x$, and BD $= (y - z)$, so that $x + (y - z) = x + y - z$.

2. When the sign immediately preceding the first bracket of a pair is negative $(-)$, the brackets may also be removed without altering the value of the expression, provided that all the positive signs within the bracket are made negative, and all the negative signs within the bracket made positive. That is to say,

$$x - (y - z) = x - y + z.$$

$$x - (y - z) \text{ really means } x - (+y - z)$$

$$= x - y + z.$$

This process may also be demonstrated in a similar manner to the above. Again, let the line AB $= x$, BC $= y$, and CD $= z$.

Draw AB or x (fig. 2) to the right, BC or y to the

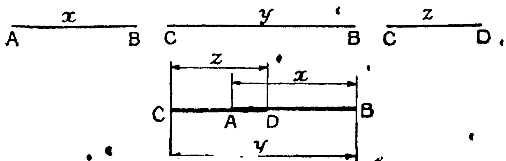

Fig. 2

left, and CD or z to the right; the result of these three is again AD.

Now, $x - (y - z) = \text{AB} - \text{DB}$

$= \text{AB} - \overbrace{\text{BC} + \text{CD}}$

$= x - y + z.$

Example 1.—Prove that

$$7 - (4 + 2) + (3 - 8) - (-24 + 3) = 17.$$

This becomes $7 - 4 - 2 + 3 - 8 + 24 - 3 = 17$ after removing all the brackets as explained. Now, collect all the positive quantities and all the negative quantities from the lower line.

$$\underbrace{7 + 3 + 24}_{34} - \underbrace{4 - 2 - 8 - 3}_{17} = 17.$$

$$34 - 17 = 17.$$

Example 2.—Simplify the expression

$$\frac{3x - 9}{3} + \frac{4x - 12}{2} - \frac{8x + 12}{4}.$$

1st step: $x - 3 + (2x - 6) - (2x + 3)$.
2nd step: $x - 3 + 2x - 6 - 2x - 3$.
3rd step: $x - 3 - 6 - 3$.
Final step: $x - 12$.

It is worth noting that the division sign, i.e. the line separating the numerator from the denominator, in $\frac{3x - 9}{3}$ acts in the same way as a bracket, because it compels one to work out each of the terms in the group as if each were enclosed in brackets.

Example 3.—Prove by removing the brackets that

$$\frac{3x + 12}{4} - \frac{2x - 4}{3} - \frac{21 - 34x}{9} = \frac{139x}{36} + 2.$$

1. Find the least common multiple of 4, 3, and 9, as in arithmetic; the L.C.M. is 36. Then say 4 into 36 = 9 for the first group, and we have $9(3x + 12)$ for the numerator. Do this for each of the remaining two groups, and the result will be

$$\frac{9(3x + 12) - 12(2x - 4) - 4(21 - 34x)}{36} = \frac{139x}{36} + 2.$$

2. Now multiply each term within the brackets by the number outside the brackets, and we obtain

$$\frac{(27x + 108) - (24x - 48) - (84 - 136x)}{36} = \frac{139x}{36} + 2.$$

3. Remove all the brackets as explained, and then obtain

$$\frac{27x + 108 - 24x + 48 - 84 + 136x}{36} = \frac{139x}{36} + 2.$$

4. For convenience only, collect all terms which contain x, and arrange as follows:

$$\frac{27x - 24x + 136x + 108 + 48 - 84}{36} = \frac{139x}{36} + 2.$$

5. Find the value of the terms involving x, as well as those which do not contain x.

$$\frac{139x + 72}{36} = \frac{139x}{36} + 2.$$

6. Cancel out 36 and 72, and we have

$$\frac{139x}{36} + 2 = \frac{139x}{36} + 2.$$

The left-hand side of the original equation has now been reduced to its simplest form, and the result proves that the two sides of the equation were equal, as indicated by the mark (=) of equality.

Exercises, with answers, on p. 108.

CHAPTER IV

SYMBOLIC NOTATION

It is most useful, and sometimes absolutely essential, to be able to convert a mathematical expression or law or rule into words, and also to be able to represent any law or rule by means of symbols. The following examples illustrate various forms, and indicate how a rule may be expressed by means of symbols; they also show how mathematical expressions may be put into words.

Example 4.—The pitch of a screw is $\frac{1}{16}$ in. How many threads are there per inch?

The pitch of a screw is the distance, centre to centre, of two neighbouring threads, as indicated in two places in fig. 3.

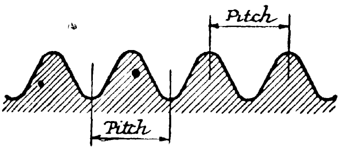

Fig. 3

In the above example there is 1 thread in $\frac{1}{16}$ in., hence

$$1 \text{ in.} \div \tfrac{1}{16} \text{ in. or } 1 \times \frac{16}{1} = 16 \text{ threads in } 1 \text{ in.}$$

The pitch is therefore the reciprocal of the threads per inch. Any two numbers of which the product is unity (1) are called reciprocal numbers. Thus 5 and $\frac{1}{5}$ are reciprocals; so are $\frac{1}{12}$ and 12; $\frac{1}{a}$ is the reciprocal

of a; $\frac{3}{4}$ is the reciprocal of $\frac{4}{3}$ or $1\frac{1}{3}$; and $\frac{x}{y}$ and $\frac{y}{x}$ are reciprocals, and so on.

In Example 4, $\frac{1}{16}$ is the reciprocal of 16; but $\frac{1}{16}$ in. is the pitch of the screw, and 16 is the number of threads per inch. These particulars may be embodied in a mathematical law:—The pitch of a screw in inches is the reciprocal of the number of threads per inch. In symbols,

$$\text{if } p = \text{the pitch in inches,}$$

$$\text{then } \frac{1}{p} = \text{the threads per inch.}$$

Further, if t = the threads per inch, the pitch will be $\frac{1}{t}$, because p and $\frac{1}{p}$, and t and $\frac{1}{t}$, are two pairs of reciprocals.

Example 5.—The fly-wheel of a mill engine makes 240 revolutions per minute (often abbreviated to 240 r.p.m.). This is equivalent to $\frac{240 \text{ revs.}}{60 \text{ secs.}} = 4$ revs. per second. How long does it take to make one revolution?

Revolutions made.	Time taken.
240	1 min. or 60 sec.
4	1 sec.
2	$\frac{1}{2}$ sec.
1	$\frac{1}{4}$ sec.

Note that $\frac{1}{4}$ is the reciprocal of 4, and 1 sec. is an interval of time.

$\frac{1}{\frac{1}{4} \text{ sec.}}$ is the reciprocal of a time interval, briefly termed a frequency. If the fly-wheel revolves once

in $\frac{1}{4}$ sec., it will revolve 4 times in 1 sec.; in other words, its frequency is 4.

Example 6.—Express V ft. per minute, in feet per second.

Since 1 sec. is $\frac{1}{60}$ part of 1 min., it follows that

$$V \text{ ft. per minute} = \frac{V}{60} \text{ ft. per second.}$$

Example 7.—A mill-engine uses S lb. of steam per hour when its load is H horse-power. How many pounds of steam will it use in D days if it runs t hr. per day? What are the meanings of the following terms: $D \times t$; $\frac{S}{H}$; $\frac{H}{S}$?

The engine uses S lb. of steam in 1 hr.

It will use $S \times t = St$ lb. of steam in t hr. per day.

,, $St \times D = StD$ lb. of steam in D days of t hr. each.

$D \times t = Dt$ = D days of t hr. each
= the total time the engine is running.

$$\frac{S}{H} = \frac{\text{pounds of steam per hour}}{\text{horse-power developed}}$$

= the pounds of steam used per hour for each horse-power developed
= the pounds of steam per horse-power hour.

This ratio $\frac{S}{H}$ is commonly used to compare the relative efficiencies of different classes and types of steam-engines.

$$\frac{H}{S} = \frac{\text{horse-power developed}}{\text{pounds of steam per hour}}$$

the horse-power developed per pound of steam per hour.

$\frac{S}{H}$ is the reciprocal of $\frac{H}{S}$, so that the lb. of steam per horse-power hour is the reciprocal of the horse-power (h.p.) developed per pound of steam per hour.

Example 8.—A loom runs at 200 picks per minute on cloth requiring 40 shots per inch. If the loom loses 15 per cent of its possible picks through unavoidable stoppages, find its production in yards per day of 9 hr.

$$200 \text{ picks per minute} \times 60 \text{ min. per hour}$$
$$= (200 \times 60) \text{ picks per hour.}$$
$$(200 \times 60) \text{ picks per hour} \times 9 \text{ hr. per day}$$
$$= (200 \times 60 \times 9) \text{ picks per day.}$$

If 15 per cent picks are lost, then 85 per cent of picks are effective, and we have

$$200 \times 60 \times 9 \times \frac{85\%}{100\%} = \text{total effective picks per day.}$$

There are 40 shots per inch, so that

$$\frac{200 \times 60 \times 9 \times 85}{100} \div 40$$
$$= \frac{200 \times 60 \times 9 \times 85}{100 \times 40} = \text{inches of cloth per day.}$$

There are 36 in. per yard, therefore

$$\frac{200 \times 60 \times 9 \times 85}{100 \times 40} \div 36$$
$$= \frac{200 \times 60 \times 9 \times 85}{100 \times 40 \times 36} = 63.75 \text{ yd. per day.}$$

In the above arithmetical example there are altogether 7 terms, 3 of which are variable for any particular cloth, the remaining 4 being constant for any particular weaving factory. The constant terms are

60, 9, and 36, representing, respectively, minutes per hour, hours per day, and inches per yard, and 100 the percentage basis.

Let y = yards of cloth per loom per day;
e = percentage of effective working time;
p = picks per minute, the loom speed;
and s = shots of weft per inch.

Then,

$$y = \frac{e \times p \times 60 \times 9}{100 \times s \times 36}$$
$$= \frac{60 \times 9 \times e \times p}{100 \times 36 \times s}$$
$$= \frac{60 \times 9}{100 \times 36} \times \frac{ep}{s}$$
$$= \frac{3}{20} \times \frac{ep}{s}$$
$$= \frac{3\,ep}{20\,s} \text{ or } \frac{\cdot 15\,ep}{s}.$$

The above numerical example can be checked by the latter expression, thus,

$$y = \frac{\cdot 15\,ep}{s}$$
$$= \frac{\cdot 15 \times 85 \times 180}{40}$$
$$= 63\cdot 75 \text{ yd. per day.}$$

Example 9.—Express in words,

$$(b \times a\,c) - \frac{a}{b\,c}.$$

The above signifies that b is to be multiplied by the product of a and c; that a is to be divided by the product of b and c; and that the value or quotient of

the second group of symbols is to be subtracted from the value of the first group contained in the brackets.

Example 10.—The picks or shots in any cloth, multiplied by the reed width in inches, multiplied by the cloth length in yards, and the product of these three terms divided by 840, is equal to the hanks of weft in a piece of cotton cloth. Express this value in symbols.

Let S = shots (or picks) per inch;
R = reed width in inches;
C = cloth length in yards;
H = hanks of weft in the cloth.

Then,

$$H = \frac{S \times R \times C}{840} = \frac{S\,R\,C}{840} \text{ hanks.}$$

Exercises, with answers, on p. 109.

CHAPTER V

RECTANGLES AND SQUARES

The two lines in a compass which cross each other and which point respectively to North and South, and to East and West, are perpendicular to each other, or at right angles to each other, because each pair of adjacent lines from the crossing-point C, in fig. 4, embraces one-quarter of the 360 degrees, or $\frac{360}{4}$ = 90 degrees. It will be evident that 90 degrees (one right angle) will be enclosed between two perpendicular lines whether such lines be of equal length or of different lengths. Thus the line EB, an extension of the line WE, is longer than

the line NS, and the line WB is considerably longer than the line NS, but all the angles in the figure are right angles. The four heavy lines which circumscribe the ring or circle are all the same length; these four heavy lines constitute a rectangle as well as a square. The dotted extension in conjunction with the east side, E, is also a rectangle, but not a square, and the complete outline of the figure in the illustration is still another but larger rectangle.

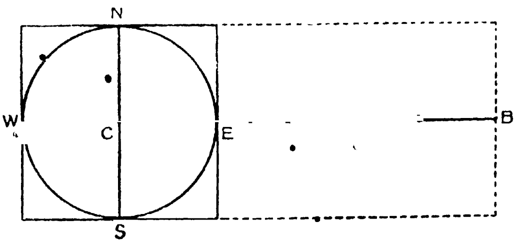

Fig 4

THE RECTANGLE.—A rectangle is thus a 4-sided figure in which all the angles are right angles, and in which two parallel sides are perpendicular to the other two parallel sides.

THE SQUARE.—A square is a 4-sided figure in which all the sides are of the same length, and all the angles are right angles.

Fig. 5 is a rectangle 6 units long and 4 units broad. It has four sides—AB, BC, CD, and DA. The side AB = the side CD, and the side BC = the side AD. The words "the angle enclosed between two adjacent sides AB and BC" may be expressed by $A\hat{B}C$. The four angles $A\hat{B}C$, $B\hat{C}D$, $C\hat{D}A$, and $D\hat{A}B$ are all right angles, that is, each angle is

equal to 90°. If AB is divided into 6 equal parts by the points 1, 2, 3, &c., and BC into 4 similar equal parts by the points 6, 7, 8, &c., and lines drawn from these points parallel to two adjacent

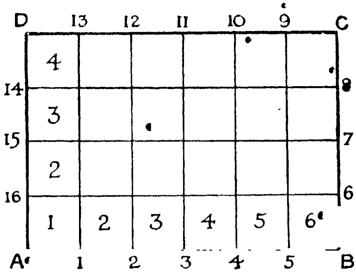

Fig 5

sides, the rectangle is divided into 4 rows of 6 little squares each, or altogether 24 little squares. Each square is 1 unit long and 1 unit broad, and is called a square unit. The whole area of the rectangle thus contains 24 sq. units.

But 6 units × 4 units = 24 sq. units; hence
length × breadth = area of rectangle.

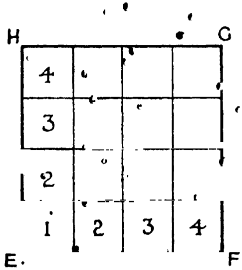

Fig 6

If the linear unit is 1 in., the unit of area will be 1 sq. in.; similarly, if the linear unit be 1 yd., the unit of area will be 1 sq. yd., and so on.

Again, with a square, such as that in fig. 6, the sides EF, FG, GH, and HE are all equal, and the 4 angles are right angles. It is clear

that each side is 4 units long, and that there are 16 sq. units in the area.

$$\begin{aligned}\text{Area of square} &= \text{length of side} \times \text{length of side}\\ &= \text{side squared, or side}^2.\end{aligned}$$

Side^2 means side multiplied by itself. In the same way we have $4^2 = 16$; $8^2 = 64$; $a^2 = a \times a$; $x^2 = x \times x$, &c.

Symbolically, let

$$\begin{aligned} A &= \text{area of rectangle;}\\ l &= \text{length of long side;}\\ b &= \text{breadth of short side.}\end{aligned}$$

Then, $A = l \times b$,

or $A = lb$.

In such an equation, A is said to be given explicitly in terms of l and b, and that l and b are only given implicitly. It is most important to note, however, that if any equation is stated in the form $A = lb$, that one can also find the values of l and b. Thus, if we divide each side of the equation $A = lb$ by b, we obtain

$$\frac{A}{b} = \frac{lb}{b}.$$

Then, by cancelling the two b's on the right hand side of the equality mark (=), we get

$$\frac{A}{b} = l,$$

$$\text{so that } l = \frac{A}{b}.$$

In words, the length of a rectangle is equal to its area divided by its breadth.

By a similar method it may be shown that

$$b = \frac{A}{l}.$$

Again, let A = the area of a square,
and s = the side of same square.

Then, as indicated above,

$$A = s^2,$$
$$\text{or } s^2 = A.$$

If the square root of each equal quantity is taken, we have

$$\sqrt{s^2} = \sqrt{A},$$
$$\text{or } s = \sqrt{A}.$$

In words, the side of a square is equal to the square root of its area.

In making use of these rules, it is essential that the units of area and length should correspond. Thus, if the square or rectangle be measured in inches, the area will be in square inches; if the area be in square yards, the length will be in yards.

A knowledge of the following tables must be acquired:—

I. Linear Measure

12 inches	=	1 foot.
3 feet	=	1 yard.
5½ yards	=	1 pole or perch.
40 poles	=	1 furlong, or 220 yards.
8 furlongs	=	1 mile, or 1760 yards, or 5280 feet.

II. Square Measure

144	square inches	=	1 square foot.
9	square feet	=	1 square yard.
$30\frac{1}{4}$	square yards	=	1 square pole.
40	square poles	=	1 rood.
4	roods, or 4840 sq. yd.	=	1 acre.

It is also useful to remember the length known as a chain (Gunter's chain). This is extensively used in land surveying, and is equal to 22 yards, or $\frac{1}{80}$th of a mile.

Example 11.—A piece of canvas is 24 in. wide and 50 yd. long. What is its area in square yards?

$$\text{Length} = 50 \text{ yd.}$$

$$\text{Breadth} = 24 \text{ in.} \div 36 \text{ in. per yard} = \frac{24}{36} \text{ yd.}$$

$$\text{Area} = l \times b$$

$$= 50 \times \frac{24}{36} = \frac{100}{3} = 33\tfrac{1}{3} \text{ sq. yd.}$$

Example 12.—A small sample of cloth measures 2 in. square and weighs 28 gr. Find the weight of the cloth in ounces per square yard. (7000 gr. in 1 lb., $437\frac{1}{2}$ gr. in 1 oz.)

$$\text{Area of sample} = \text{side}^2$$

$$= 2 \times 2 = 4 \text{ sq. in.}$$

$$\text{Area of 1 yd.} = 36 \times 36 = 1296 \text{ sq. in.}$$

$$\frac{28 \text{ gr.}}{4 \text{ sq. in.}} = \text{weight of 1 sq. in.}$$

$$\therefore \text{Weight of 1 sq. yd.} = \frac{28 \times 1296}{4} \text{ gr.}$$

$$= \frac{28 \times 1296}{4 \times 437\frac{1}{2}} \text{ oz.}$$

$$= 20{\cdot}736 \text{ oz.,}$$

$$\text{or weight of cloth} = 20{\cdot}736 \text{ oz. per square yard.}$$

Example 13.—A cotton carding-room measures 108 ft. long by 48 ft. wide. It contains 36 carding-engines each occupying a floor space of 10 ft. 4 in. by 6 ft. Find the area of the floor, the area actually occupied by the machinery, and the area taken up for passes and the like.

$$\begin{aligned}
\text{Area of room} &= \text{length} \times \text{breadth} \\
&= 108 \text{ ft.} \times 48 \text{ ft.} \\
&= 5184 \text{ sq. ft.} \\
\text{Floor space for 1 engine} &= 10\tfrac{1}{3} \text{ ft.} \times 6 \text{ ft.} \\
&= 62 \text{ sq. ft.} \\
\text{Area occupied by 36 engines} &= (62 \times 36) \text{ sq. ft.} \\
&= 2232 \text{ sq. ft.} \\
\text{Area left for passes} &= (5184 - 2232) \text{ sq. ft.} \\
&= 2952 \text{ sq. ft.}
\end{aligned}$$

Example 14.—Find the cost of paving a passage in a factory, 88 ft. long by 9 ft. wide, at the rate of 47*s.* per square yard.

$$\begin{aligned}
\text{Area of passage} &= \text{length} \times \text{breadth} \\
&= 88 \text{ ft.} \times 9 \text{ ft.} \\
&= (88 \times 9) \text{ sq. ft.} \\
&= \frac{88 \times 9}{9 \text{ sq. ft. per sq. yd.}} = 88 \text{ sq. yd.} \\
\text{Cost of paving} &= 88 \text{ sq. yd.} \times 47s. \text{ per sq. yd.} \\
&= 4136s., \text{ or } \pounds 206,\ 16s.
\end{aligned}$$

Example 15.—Find the cost of a carpet, 27 in. wide, at 10*s.* 6*d.* per yard, for the complete covering of a room 22 ft. long by 18 ft. wide.

$$\begin{aligned}
\text{Area of carpet} &= \text{area of floor} \\
&= 22 \text{ ft.} \times 18 \text{ ft.} \\
&= (22 \times 18) \text{ sq. ft.} \\
\text{Width of carpet} &= 27 \text{ in.} \\
&= 2\tfrac{1}{4} \text{ ft.}
\end{aligned}$$

$$
\begin{aligned}
\text{Length of carpet} &= (22 \times 18) \text{ sq. ft.} \div 2\tfrac{1}{4} \text{ ft.} \\
&= 176 \text{ ft.} \\
&= 58\tfrac{2}{3} \text{ yd.} \\
\text{Cost of carpet} &= 58\tfrac{2}{3} \text{ yd., at } 10s.\ 6d. \text{ per yard,} \\
&= \frac{176}{3} \times \frac{21}{2} \text{ shillings} \\
&= 616 \text{ shillings, or } £30,\ 16s.
\end{aligned}
$$

Example 16.—A wool-store is to be erected on a square plot of ground known to contain 3364 sq. ft. Find the length of its sides.

$$
\begin{aligned}
\text{Area of square} &= \text{side}^2, \\
\text{and side} &= \sqrt{\text{area}} \\
&= \sqrt{3364} \\
&= 58 \text{ ft.}
\end{aligned}
$$

Exercises, with answers, on p. 110.

CHAPTER VI

TRIANGLES

As the name implies, a triangle is a plane figure bounded by three straight lines and including three angles. The definitions regarding the various types of triangles appear below.

EQUILATERAL TRIANGLE.—An equilateral triangle has its three sides equal, as ABC in fig. 7. It may be proved (*Euc.* I, 5) that the three angles of this triangle are equal, and since the three angles of any triangle are equal to 180°, each angle of an equilateral triangle is 60°.

ISOSCELES TRIANGLE.—An isosceles triangle has two equal sides, as DF and FE in triangle DEF in

fig. 7. It may be proved (*Euc.* I, 5) that the angles opposite the equal sides are equal; hence, if one angle is known, the others may easily be found.

SCALENE TRIANGLE.—A scalene triangle has three unequal sides, as GHK in fig 7.

All the above types are named with reference to the

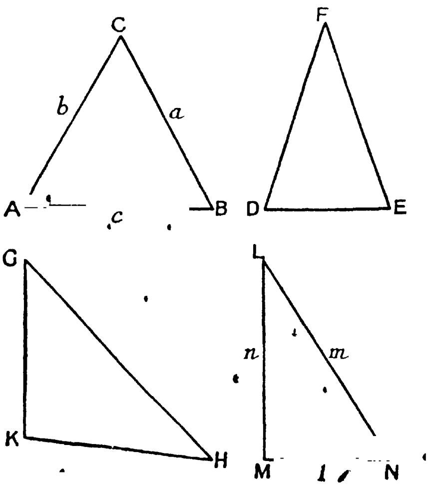

Fig. 7

lengths of the sides. Triangles may also be classified according to the size of the angles they contain, and as follows:—

RIGHT-ANGLED TRIANGLE.—A right-angled triangle has one of its angles a right angle (90°), as in the triangle LMN in fig 7. The angle M is a right angle, and the side LN opposite the right angle is called the hypotenuse. It is proved in *Euc.* I, 47, that the

square on the hypotenuse is equal to the sum of the squares on the other two sides, i.e. $m^2 = l^2 + n^2$.

Obtuse-angled Triangle.—An obtuse-angled triangle has one of its angles obtuse, i.e. greater than 90°, as angle K in triangle GKH, fig. 7.

Acute-angled Triangle. An acute-angled triangle has all its angles acute, i.e. each is less than 90°, as ABC or DEF, fig. 7.

The right-angled triangle is a very important one in all branches of work; its properties enable a certain class of problem to be easily solved. The word triangle may be conveniently represented by the sign $\triangle$.

Example 17.—Find the length of the hypotenuse of a right-angled triangle when the other two sides are respectively 9 ft. and 12 ft.

Referring to fig. 7, in the $\triangle$ LMN,

$$\begin{aligned} m^2 &= l^2 + n^2, \\ \therefore m^2 &= 9^2 + 12^2 \\ &= 81 + 144 \\ &= 225, \\ \text{and } m &= \sqrt{225} \\ &= 15 \text{ ft., the length of hypotenuse.} \end{aligned}$$

It should also be noted that if

$$m^2 = l^2 + n^2,$$

and if l^2 be subtracted from each side of the equation,

$$\text{then } m^2 - l^2 = n^2,$$
$$\text{whence } n = \sqrt{m^2 - l^2}.$$

In the same way it may be shown that

$$l = \sqrt{m^2 - n^2}.$$

Notice that $9 + 16 = 25$,
or $3^2 + 4^2 = 5^2$.

Consequently, it follows that a right-angled triangle may always be drawn if the three sides are kept in the proportion of 3, 4, and 5. Many textile machines are in two or more separate parts; to take a simple example, a beaming-machine consists, say, of a bobbin creel or bank and the winding-on mechanism.

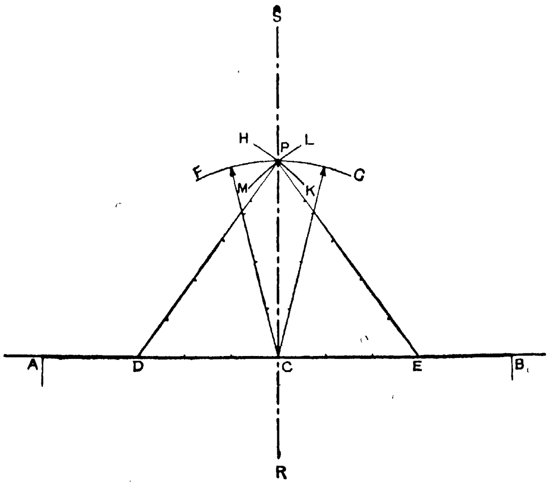

Fig. 8

In order that each part may work properly with the other, it is essential that they should have a common centre-line, i.e. the centre-line of each part should lie in one straight line. Thus, in fig. 8, AB represents the back line of the beaming-machine proper transferred to the floor, and C is the middle point. From C mark off to left and to right the points D and E, each 3 units from C. The units would probably be 1 ft., but any other suitable unit may be utilized.

With C as centre, and a radius of 4 units, draw the arc FG. With D as centre, and a radius of 5 units, draw the arc HK, and with the same radius and E as centre, draw the arc LM. If correctly drawn, the three arcs will meet at a common point P. Join C to P, and extend the line in the required direction to S and R.

In the triangles PCD and PCE, CD and CE are each 3 units, PD and PE are each 5 units, and the common side CP is 4 units. The triangles are therefore right-angled, the right angles being $P\hat{C}D$ and $P\hat{C}E$. The line RS will thus coincide with the centre-line of all the parts of the machine.

The following three statements concerning triangles are proved by Euclid, and are worth keeping in mind:—

1. Any two sides of a triangle are together greater than the third side. (*Euc.* I, 20.)

2. The three angles of a triangle are together equal to two right angles. $2 \times 90° = 180°$. (*Euc.* I, 32.)

3. The area of a triangle is half that of the rectangle on the same base and having the same altitude. (*Euc.* I, 41.)

Any of the three points or apices of a triangle may be called the vertex; the side opposite the vertex then becomes the base. In fig. 9, the point A is the vertex of the $\triangle$ABC, and BC is the base; similarly, G is the vertex of the $\triangle$GBC. The perpendicular AF is drawn from the vertex A of the first triangle to the base, so that $B\hat{F}A = A\hat{F}C = 90°$, and the line AF measures the height or altitude of the triangle. The line ED is parallel to the line BC, and hence the rectangle EBCD is on the same base, BC, as the

$\triangle$ABC, and the altitude AF = the side BE. It is not difficult to see, and it may easily be proved, that the two shaded triangles are equal in area to

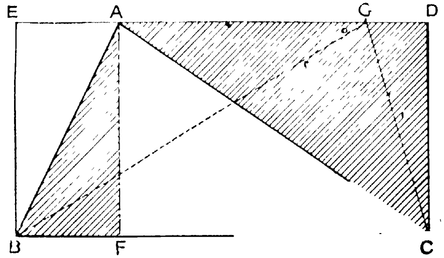

Fig. 9

the two unshaded triangles, and therefore $\triangle$ABC = $\frac{1}{2}$ rect. EBCD. Consequently,

$$\begin{aligned} \text{Area of } \triangle ABC &= \tfrac{1}{2} \text{ area of rect. EBCD} \\ &= \tfrac{1}{2} (BC \times EB) \\ &= \frac{BC \times AF}{2} \\ &= \frac{ab}{2}, \end{aligned}$$

where a = the altitude, and b = the base.

Let A = the area of the triangle,

then $A = \dfrac{ab}{2}$.

Multiply both sides of the equation by 2, then $2A = ab$.

Divide both sides of the equation by b, then $\dfrac{2A}{b} = a$,

or $a = \dfrac{2A}{b}$;

that is to say, the altitude of a triangle is equal to twice the area divided by the base.

In a similar way, it may be shown that

$$b = \frac{2A}{a},$$

i.e. the base of a triangle is equal to twice the area divided by the altitude.

Example 18.—Find the weight of a triangular sheet of metal intended for a sliver conductor if the base measures 6 ft. 11 in. and the altitude is 3 ft. 6 in., the sheet weighing 8 oz. per square foot.

$$\begin{aligned}
\text{Area} &= \frac{ab}{2} \\
&= \frac{3\text{ ft. }6\text{ in.} \times 6\text{ ft. }11\text{ in.}}{2} \\
&= \frac{42\text{ in.} \times 83\text{ in.}}{2 \times 144}\text{ sq. ft.} \\
&= 12 \cdot 104\text{ sq. ft.} \\
\text{Weight} &= 12 \cdot 104\text{ sq. ft.} \times 8\text{ oz.} \\
&= \frac{12 \cdot 104 \times 8}{16\text{ oz. per pound}} \\
&= 6 \cdot 052\text{ lb.}
\end{aligned}$$

In practice, it is not always possible to obtain the vertical height; the following method illustrates a case where it is more convenient to measure the three sides only. It is a common and excellent method to indicate the lengths of the sides of a triangle by small letters which correspond to the similar larger letters at the opposite vertices. This is done in two of the triangles in fig. 7.

Let s = the semi-perimeter of a triangle,
i.e. half the sum of its 3 sides.
a, b, and c = the 3 sides.

$$\text{Then, } 2s = a + b + c,$$
$$\text{and } s = \frac{a + b + c}{2}.$$

It can be proved that the area A of a triangle is

$$A = \sqrt{s(s-a)(s-b)(s-c)}.$$

Example 19.—Find the area of a triangular plot of ground intended for drying bleached hanks of yarn, the sides of the plot measuring 40 yd., 46 yd., and 50 yd. respectively.

$$\begin{aligned}
\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)}, \\
\text{and } s &= \tfrac{1}{2}(a + b + c) \\
&= \tfrac{1}{2}(40 + 46 + 50) = 68 \\
\therefore A &= \sqrt{68(68-40)(68-46)(68-50)} \text{ sq. yd.} \\
&= \sqrt{68 \times 28 \times 22 \times 18} \text{ sq. yd.} \\
&= \sqrt{753984}, \text{ or } 868 \cdot 3 \text{ sq. yd.}
\end{aligned}$$

Exercises, with answers, on p. 111.

CHAPTER VII

QUADRILATERALS

QUADRILATERAL.—The term quadrilateral may be applied to any plane figure bounded by four straight lines. The square and rectangle are thus special kinds of quadrilaterals. Other special forms of quadrilateral are the parallelogram, the rhombus, the trapezium, and the trapezoid.

PARALLELOGRAM.—A parallelogram is a quadrilateral figure in which both pairs of opposite sides are parallel. The square and rectangle are therefore quadrilateral figures, or rather parallelograms; in

general, a parallelogram is considered to have angles which are not right angles. Examples appear at ABCD and EFGH in fig. 10.

RHOMBUS.—A rhombus is a quadrilateral figure all the four sides of which are equal, and all the angles differing from right angles. EFGH, fig. 10, is therefore a rhombus.

The straight lines AC, BD, KM, LN, &c., drawn

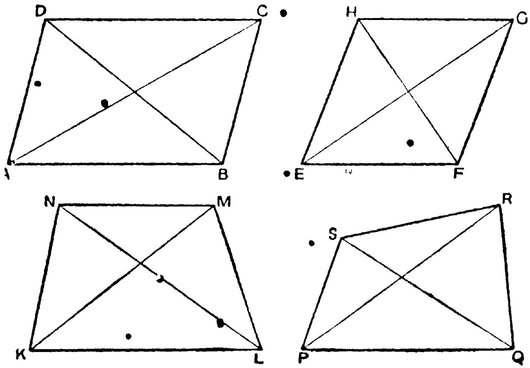

Fig. 10

from corner to corner in fig. 10 are called diagonals. Note that each diagonal divides the quadrilateral into two equal or unequal triangles.

TRAPEZIUM.—A trapezium is a quadrilateral which has one pair of opposite sides parallel, as KL and NM in the diagram KLMN, fig. 10.

TRAPEZOID. — A trapezoid is a quadrilateral in which no two sides are parallel, as PQRS, fig. 10.

N.B.—These two definitions are often reversed.

Referring to fig. 10, it is evident from the construc-

tion of the two upper figures, and is proved in *Euc.* I, 34, that:—

1. The opposite sides of a parallelogram or rhombus are equal.

2. The opposite angles of a parallelogram or rhombus are equal.

3. The diagonals of a parallelogram or rhombus bisect one another, i.e. they divide each other into two equal parts, and also divide the figure into two triangles which are equal in every respect.

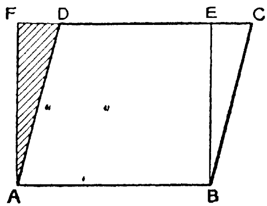

Fig. 11

It may also be shown that the diagonals of a rhombus bisect each other at right angles.

All the rules which are applicable to the solution of triangles may also be applied to the solution of quadrilaterals, since the latter can be divided into two triangles. Special rules are, however, often much more convenient than the above method.

Fig. 11 illustrates a parallelogram ABCD on a base AB; this line AB is also the base of the rectangle ABEF.

It may be proved (*Euc.* I, 35) that the area of a parallelogram is equal to that of a rectangle on the same base and having the same height. If $\triangle$ BCE is cut off and placed in a position so as to form

$\triangle$ ADF (shown shaded in fig. 11), a rectangle ABEF is formed. Then,

Area of parallelogram ABCD = area of rectangle ABEF

= AB × AF

= base × perpendicular height.

If A = area of parallelogram,

b = base,

h = perpendicular height,

then A = bh,

from which it may be deduced that,

$$b = \frac{A}{h},$$

$$\text{and } h = \frac{A}{b}.$$

Example 20.—Find the area of a parallelogram the base of which is 4 ft. 3 in. and the perpendicular height (the height at right angles to the base) is 6 ft. 2 in.

Area A = bh.

= 51 in. × 74 in.

= 3774 sq. in.

= 26·2 sq. ft.

Example 21.—Find the height of a rhombus the area of which is 18 sq. ft., and the side 4 ft. 6 in.

Area A = $b \times h$,

whence $h = \frac{A}{b}$

$$= \frac{18 \text{ sq. ft.}}{4 \text{ ft. } 6 \text{ in.}} = \frac{18}{4\frac{1}{2}} = 4 \text{ ft.}$$

TRAPEZIUM.—In fig. 12, KLMN represents a trapezium with the base KL parallel to the opposite

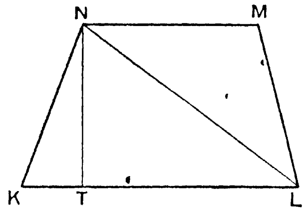

Fig. 12

side MN. Join LN, and draw NT perpendicular to KL; then $N\hat{T}K$ and $N\hat{T}L$ are right angles. Now,

$$\begin{aligned} \text{Area of trapezium KLMN} &= \text{area of } \triangle\text{KLN} + \text{area of } \triangle\text{LMN} \\ &= \frac{\text{KL} \times \text{NT}}{2} + \frac{\text{MN} \times \text{NT}}{2}. \end{aligned}$$

Taking the common factor $\frac{\text{NT}}{2}$ outside, we have

$$\begin{aligned} \text{A} &= \frac{\text{NT}}{2}(\text{KL} + \text{MN}) \\ &= \text{NT}\frac{(\text{KL} + \text{MN})}{2}. \end{aligned}$$

But KL and MN are the two parallel sides of the trapezium, so that

$$\frac{\text{KL} + \text{MN}}{2} = \text{half the sum of the parallel sides,}$$

and NT is the perpendicular distance between them, so that

Area of trapezium = $\frac{1}{2}$ sum of parallel sides × the perpendicular distance between them.

If x and y are the parallel sides, and d is the perpendicular distance between x and y, then, since

$$A = \text{the area of the trapezium,}$$
$$A = d\left(\frac{x+y}{2}\right),$$

from which it may be deduced that

$$d = \frac{2A}{x+y}.$$

Example 22.—A factory or mill site is in the form of a trapezium; the two parallel sides measure 6 chains and 8 chains respectively, and the perpendicular distance between them is 5 chains. Find the cost of the site at the rate of £750 per acre.

Note: 22 yd. = 1 chain.
22^2 or 484 sq. yd. = $\frac{1}{10}$ ac.

$$\begin{aligned}
\text{Area of site} &= \text{area of trapezium} \\
&= d\,\frac{(x+y)}{2} \\
&= 5\,\frac{(6+8)}{2} \\
&= 5 \times 7 \\
&= 35 \text{ sq. chains} \\
&= 3\cdot 5 \text{ ac.} \\
\text{Price of site} &= £750 \times 3\cdot 5 \text{ ac.} \\
&= £2625.
\end{aligned}$$

IRREGULAR QUADRILATERALS.—The area of an irregular quadrilateral or trapezoid, such as PQRS in fig. 10, is found by drawing a diagonal, and calculating the areas of the two triangles thus formed; any measurements which enable the areas of the two triangles to be found will also be necessary to calculate the area of the quadrilateral. The most convenient way involves only three measurements, and

may be termed the off-set method; it is explained below.

Example 23.—Find the rent, at £5 per acre, of a quadrilateral plot of ground of which one diagonal measures 15 chains, and the off-sets from it to the other two vertices are 8 chains and 12 chains respectively.

Referring to fig 13, ABCD is a quadrilateral representing the plot; the diagonal AC is 15 chains; FD

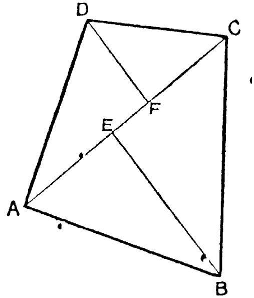

Fig 13

and EB are the two off-sets of 8 and 12 chains, both being perpendicular to the diagonal AC.

$$\text{Let AC} = d;$$
$$\text{FD} = a;$$
$$\text{EB} = b.$$

Area of quadrilateral ABCD = area of $\triangle$ACD + area of $\triangle$ABC.

$$\begin{aligned} A &= (\tfrac{1}{2}\,\text{AC} \times \text{FD}) + (\tfrac{1}{2}\,\text{AC} \times \text{EB}) \\ &= \left(\frac{d}{2} \times a\right) + \left(\frac{d}{2} \times b\right) \\ &= \frac{d}{2}(a + b) \\ &= \tfrac{1}{2}\ \text{diagonal} \times \text{the sum of the off-sets.} \end{aligned}$$

With the values given in Example 23, we have:

$$\begin{aligned} A &= \frac{15}{2}(8 + 12) \\ &= \frac{15}{2} \times 20 \\ &= 150 \text{ sq. chains} \\ &= 15 \text{ ac.} \\ \text{Rent} &= 15 \times £5 = £75. \end{aligned}$$

Exercises, with answers, on p. 113.

CHAPTER VIII

POLYGONS

POLYGON.—A polygon is a plane figure bounded by more than 4 straight lines. A regular polygon is one in which all the sides are equal and all the angles equal.

A polygon with 5 sides is called a Pentagon.
,, ,, 6 ,, ,, Hexagon.
,, ,, 7 ,, ,, Heptagon.
,, ,, 8 ,, ,, an Octagon.
,, ,, 9 ,, ,, a Nonagon.
,, ,, 10 ,, ,, Decagon.

ABCDEFGH in fig. 14 is a regular octagon, since all its 8 sides are equal and all its angles are equal. If the angles are bisected, the bisectors (shown in thin lines from the centre of the circle to the outer circle) will all meet at one point, the centre, as stated. With a radius equal to half the diameter, i.e. half of AE, and rotating about the common central point, a circle may be drawn which will touch all the points

ABCDEFGH; this outer circle is called the circumscribed circle.

The dotted lines which bisect the angles of the 8 isosceles triangles also bisect the 8 sides of the octagon. If a circle is drawn with a radius equal to the distance from the common central to point the middle of any of the sides of the octagon, this circle will pass through,

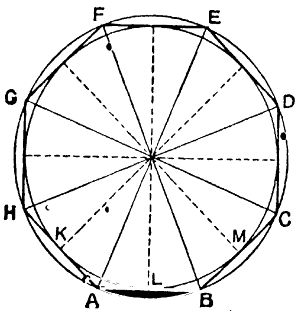

Fig 14

or touch, all the points K, L, M, &c. Such an inner circle is called the inscribed circle.

Since the octagon is divided up into 8 equal isosceles triangles, the area of the octagon may be found by calculating one of the triangles and multiplying the result by 8. The general case may be stated as follows. Let the polygon have n sides, then: Area of polygon = the area of n equal triangles.

$$A = n \times \frac{ab}{2} \text{ (where } a = \text{altitude, and } b = \text{base)}$$

$$= \left(\frac{n}{2} \times b\right) a.$$

But since $(n \times b)$ = the number of sides multiplied

by the length of each, the result is the perimeter of the polygon. Hence

$$\left(\frac{n}{2} \times b\right) = \frac{1}{2} \text{ the perimeter,}$$

so that the rule deduced may be stated as follows:—

To find the area A of a regular polygon of n sides, multiply half the perimeter by the perpendicular drawn from the centre of a side to the centre of the circumscribed and inscribed circles.

Example 24.—Find the cost of a hexagonal sheet of metal, 16 in. side, at the rate of 4*s.* per square foot; the perpendicular which bisects the side and passes to the centre is $13\frac{7}{8}$ in.

$$\text{Area} = \frac{nab}{2},$$

$$A = \frac{6 \times 13\frac{7}{8} \times 16}{2} \text{ sq. in.}$$

$$= \frac{6 \times 13{\cdot}875 \times 16}{2 \times 144} \text{ sq. ft.}$$

$$= 4{\cdot}625 \text{ sq. ft.}$$

$$\text{Cost} = 4{\cdot}625 \text{ sq. ft.} \times 4s. \text{ per square foot}$$

$$= 18s.\ 6d.$$

It is not always convenient in practice to measure the perpendicular; difficulty may be experienced in finding the exact centre of the circumscribed and inscribed circles. Rules can be deduced, however, which are independent of the perpendicular, but these involve trigonometrical ratios of the angles of the triangle; the discussion of these ratios is outside the scope of the present chapter.

When the areas of irregular polygons are required, it is always possible to divide such figures into a number of triangles, rectangles, trapeziums, or the

like, or into a number of each kind. Such a case is demonstrated in the next example.

Example 25.—A plan of a dining-room is repro-

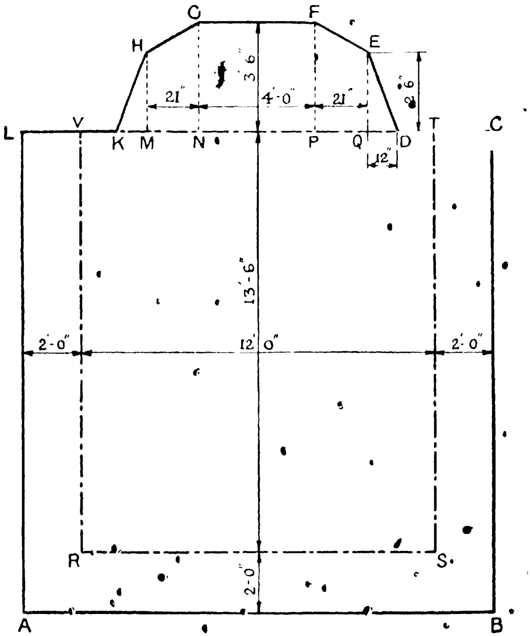

Fig. 15

duced in fig. 15. Part of the floor is covered with a carpet RSTV, 4 yd. by $4\frac{1}{2}$ yd., the remainder being covered with linoleum. Find the whole area of the room, that of the carpet, and that covered by linoleum.

Area of floor = 2 unequal rectangles ABCL and NPFG,
+ 2 equal triangles KMH and DQE,
+ 2 equal trapeziums MNGH and PQEF.

$$A_f = (16 \text{ ft.} \times 15 \text{ ft. } 6 \text{ in.}) + (4 \text{ ft.} \times 3 \text{ ft. } 6 \text{ in.})$$
$$+ 2\left(\frac{12 \text{ in.}}{2} \times 2 \text{ ft. } 6 \text{ in.}\right) + 2\left(\frac{2 \text{ ft. } 6 \text{ in.} + 3 \text{ ft. } 6 \text{ in.}}{2} \times 21 \text{ in.}\right)$$
$$= (16 \times 15\tfrac{1}{2}) + (4 \times 3\tfrac{1}{2}) + 2(\tfrac{1}{2} \times 2\tfrac{1}{2}) + 2(3 \times 1\tfrac{3}{4})$$
$$= (248 + 14 + 2\tfrac{1}{2} + 10\tfrac{1}{2}) \text{ sq. ft.}$$
$$= 275 \text{ sq. ft.}$$

Area of carpet = 13 ft. 6 in. × 12 ft.
$A_c = (13\tfrac{1}{2} \times 12)$ sq. ft.
= 162 sq. ft. of carpet.

Area of linoleum = 275 sq. ft. − 162 sq. ft.
$\therefore A_l$ = 113 sq. ft.

NOTE.—The subscript letters *f*, *l*, and *c* in A_f, A_l, and A_c must not be confused with such expressions as A^f, A^l, and A^c. In the former cases, the small letters are distinguishing signs; in the latter cases, the small letters are indices. Indices are fully explained in Chap. XX.

Exercises, with answers, on p. 113.

CHAPTER IX

CIRCLE

CIRCLE.—A circle is a plane figure enclosed by a curved line called the circumference; all points on this circumference are equidistant from the centre, and this distance is equal to a radius. ABD, fig. 16,

shows three points on the circumference of a circle, and C indicates the central point. It will thus be seen that the lines CA, CB, and CD are three radii of this circle. Moreover, two of these radii, CA and CD, are in one straight line; the two radii, so disposed, form a diameter of the circle; hence, AD is a diameter, and all other straight lines which touch two points on the circumference and pass through the

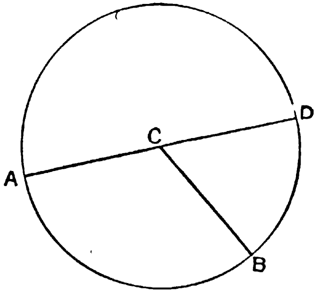

Fig. 16

centre C are diameters. Circles are often measured in terms of their radii, but in practice it is perhaps more common to measure them in terms of their diameters.

It is found that the ratio between the circumference and the diameter of a circle is constant. Thus,

$$\frac{\text{Circumference}}{\text{Diameter}} = \text{approximately } 3\tfrac{1}{7} \text{ or } \tfrac{22}{7}.$$

The exact value of the ratio between the circumference and the diameter cannot be expressed in figures, although it may, of course, be written with any required degree of accuracy. The above value, $3\frac{1}{7}$, is roughly correct; the value $3 \cdot 14$ is more correct, and is often used; $3 \cdot 142$ is correct to three places of

decimals; 3·1416 is also often used, and is correct to four places of decimals; still more correct values are: 3·14159, 3·141593, and 3·1415926. This ratio in mathematics is generally expressed by the Greek letter π (pronounced *pi*); hence, one may write

$$\frac{\text{Circumference of circle}}{\text{Diameter of circle}} = \pi,$$

where $\pi = \frac{22}{7}$, 3·14, 3·1416, &c., according to the degree of accuracy required.

Let c = the circumference of a circle;
r = the radius;
d = the diameter;
π = the above-mentioned ratio.

Then, $$\frac{c}{d} = \pi. \quad \ldots\ldots (1)$$

or $$c = \pi d. \quad \ldots\ldots (2)$$

Dividing each side of the latter equation by π, we have

$$\frac{c}{\pi} = \frac{\pi d}{\pi},$$

whence $$d = \frac{c}{\pi} \quad \ldots\ldots (3)$$

By definition, a diameter equals 2 radii, hence,

$$d = 2r \quad \ldots\ldots (4)$$

It is occasionally convenient to use the radius instead of the diameter, so, substituting $2r$ for d in equations (1), (2), and (3), it is found that:

1. $\frac{c}{2r} = \pi \quad \ldots\ldots (5)$
2. $c = \pi 2r$, usually stated $2\pi r \quad \ldots\ldots (6)$
3. $2r = \frac{c}{\pi}$, or $r = \frac{c}{2\pi} \quad \ldots\ldots (7)$

Example 26.—Find the circumference of the take-up roller of a loom if the effective diameter of the roller is $5\frac{1}{2}$ in. (Use $\pi = 3{\cdot}14$.)

$$c = \pi d$$
$$= 3{\cdot}14 \times 5\tfrac{1}{2} = 17{\cdot}27 \text{ in.}$$

Example 27.—Two shafts are connected by means

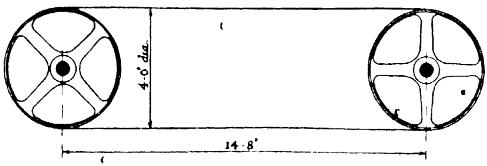

Fig. 17

of two pulleys and a belt. See fig. 17. Find the minimum length of belt required.

$$\begin{aligned}
\text{Length} &= 2(\tfrac{1}{2} \text{ pulley circumference}) \\
&\qquad + 2(\text{distance between centres}) \\
&= \text{pulley circumference} + (2 \times 14 \text{ ft. } 8 \text{ in.}) \\
&= \pi d + 29 \text{ ft. } 4 \text{ in.} \\
&= (3{\cdot}14 \times 4) + 29 \text{ ft. } 4 \text{ in.} \\
&= 12{\cdot}56 \text{ ft.} + 29{\cdot}33 \text{ ft.} \\
&= 41{\cdot}89 \text{ ft.} \doteq 41 \text{ ft. } 10\tfrac{1}{2} \text{ in.}
\end{aligned}$$

In this example and the next no allowance has been made for the thickness of the belt.

Example 28.—A double-beater scutcher is driven by means of a 10-in.-diameter pulley running at 1200 revolutions per minute. Find the speed of the belt in feet per minute.

In 1 revolution of the pulley the belt passes through a distance equal to the circumference of the pulley.

$$\text{Circumference of pulley} = \pi d$$
$$= 3 \cdot 14 \times 10 \text{ in.}$$
$$= \frac{3 \cdot 14 \times 10}{12} \text{ ft.}$$

In 1 min. the pulley makes 1200 revolutions;

$$\therefore \text{speed of belt} = \frac{3 \cdot 14 \times 10}{12} \times 1200$$
$$= 3140 \text{ ft. per minute.}$$

Example 29.—The cylinder of a carding-engine is 50 in. diameter over the wire, and runs at 165 revolutions per minute. Find the surface speed of the pins in feet per minute.

Surface speed = circumference of roller in feet × revolutions per minute.

$$S = \pi d \times \text{r.p.m.}$$
$$= [(3 \cdot 14 \times 50) \times 165] \text{ in. per minute}$$
$$= \frac{3 \cdot 14 \times 50 \times 165}{12} \text{ ft. per minute}$$
$$= 2158 \cdot 75 \text{ ft. per minute.}$$

Example 30.—It is desired to check the diameter of a fly-wheel by measuring its circumference. A steel tape is passed round the wheel and indicates 37 ft. 8½ in. If the diameter is nominally 12 ft., what is the error?

$$d = \frac{c}{\pi}$$
$$= \frac{37 \text{ ft. } 8\tfrac{1}{2} \text{ in.}}{3 \cdot 1416} = \frac{452 \cdot 5}{3 \cdot 1416} \text{ in.}$$
$$= 144 \cdot 035 \text{ in.} = 12 \text{ ft. } 0 \cdot 035 \text{ in.}$$

The fly-wheel is therefore 0·035 in. (fully $\frac{1}{32}$) too large in diameter.

Example 31.—An emery-wheel, 10-in. diameter, is to have a grinding speed of 5000 ft. per minute. How many r.p.m. should it make?

$$\text{Surface speed} = \pi d \times \text{r.p.m.},$$
$$\text{or } S = \pi d \times \text{r.p.m.};$$
$$\text{r.p.m.} = \frac{S}{\pi d}$$
$$= \frac{5000}{3\cdot14 \times 10 \text{ in.}}$$
$$= \frac{5000 \times 12}{3\cdot14 \times 10 \text{ in.}} = 1910\cdot8 \text{ r.p.m.}$$

AREA OF A CIRCLE.—Referring to fig. 14, where a regular octagon (8-sided polygon) is shown, let it be assumed that the number of sides within the circumscribed circle be increased to 16; then it is clear that the length of each side would become much shorter than those shown, and that the lines would appear nearer to the circumscribed circle; as the number of sides is further increased, they approach nearer and nearer to the outer circle, while if the number of sides is increased to a sufficiently great degree, it will become impossible to distinguish between the polygon and the circumscribed circle. A circle may thus be regarded as a polygon with an infinitely large number of sides, and hence the rule for finding the area of a regular polygon may also be applied to the circle.

$$\text{Area of circle} = \text{area of polygon},$$
$$A = \tfrac{1}{2}\text{ perimeter} \times \text{perpendicular}$$
$$= \tfrac{1}{2}\text{ circumference} \times \text{radius}$$
$$= \tfrac{1}{2}(2\pi r) \times r$$
$$= \pi r^2.$$

Again, since a diameter is twice the radius, $r = \frac{d}{2}$.

$$\therefore \text{Area of circle} = \pi r^2$$
$$= \pi\left(\frac{d}{2}\right)^2$$
$$= \pi\frac{d^2}{2^2}$$
$$= \pi\frac{d^2}{4},$$

or, as it is commonly expressed, $\frac{\pi}{4}d^2 = \cdot 7854 d^2$.

The area of a circle may therefore be expressed by the formula

$$A = \cdot 7854 d^2.$$

Dividing each side of the equation by ·7854, we have

$$\frac{A}{\cdot 7854} = d^2,$$

$$\text{hence } d = \frac{\sqrt{A}}{\sqrt{\cdot 7854}} = \sqrt{\frac{A}{\cdot 7854}}.$$

The equation may also be expressed in terms of the radius. Thus,

$$\text{Area} \quad A = \pi r^2,$$
$$\text{whence } r^2 = \frac{A}{\pi},$$
$$r = \sqrt{\frac{A}{\pi}}$$
$$= \sqrt{\frac{A}{3\cdot 1416}}.$$

Example 32.—Find the area of the 10-in.-diameter piston of a gas-engine.

$$
\begin{aligned}
\text{Area A} &= \frac{\pi}{4}d^2 \\
&= \cdot 7854d^2 \\
&= \cdot 7854 \times 10 \times 10 \text{ sq. in.} \\
&= 78\cdot 54 \text{ sq. in.}
\end{aligned}
$$

Example 33.—The total pressure on the face of the piston of a steam-engine at a certain point of its stroke is 16,000 lb. If the piston is 20-in. diameter, find the pressure per square inch.

Area of face of piston

$$
\begin{aligned}
\text{or A} &= \frac{\pi}{4}d^2 \\
&= \cdot 7854 \times 20 \times 20 \text{ sq. in.} \\
&= 314\cdot 16 \text{ sq. in.}
\end{aligned}
$$

$$
\text{Pressure per sq. in.} = \frac{\text{Total pressure in pounds}}{\text{Area in square inches}}.
$$

$$
\therefore \text{P} = \frac{16000}{314\cdot 16} = 50\cdot 93 \text{ lb. per square inch.}
$$

Example 34.—The plunger of a pump used in size or starch mixing has a lifting area of 12·57 sq. in. What is its diameter?

$$
\begin{aligned}
d &= \sqrt{\frac{\text{A}}{\cdot 7854}} \\
&= \sqrt{\frac{12\cdot 57}{\cdot 7854}} \\
&= \sqrt{16\cdot 004} = 4 \text{ in.}
\end{aligned}
$$

Certain problems may be solved by finding the difference between the areas of two circles. In this connection special methods are employed which often save time and trouble.

Notice first that

$$16 = 4^2, \text{ since } 4 \times 4 = 16,$$
$$\text{and } 9 = 3^2, \text{ since } 3 \times 3 = 9.$$
$$\text{Now, } (16 - 9) = 7.$$
$$\therefore (4^2 - 3^2) = 7.$$
$$\text{Also, } (4 + 3)(4 - 3) = 7,$$
$$\text{since } 7 \times 1 = 7.$$
$$\text{Again, } 144 = 12^2, \text{ since } 12 \times 12 = 144,$$
$$\text{and } 81 = 9^2, \text{ since } 9 \times 9 = 81.$$
$$144 - 81 = 63.$$
$$\therefore 12^2 - 9^2 = 63.$$
$$\text{Now, } (12 + 9)(12 - 9) = 63,$$
$$21 \times 3 = 63.$$

The above is true in every case, and the process may be stated generally as under:—

The difference of the squares of any two quantities is equal to the product of the sum and the difference of the two quantities. Let the two quantities be represented symbolically by the letters A and B; then

$$A^2 - B^2 = (A + B)(A - B).$$

If x and y are the quantities, then

$$x^2 - y^2 = (x + y)(x - y);$$

$(x + y)$ and $(x - y)$ are said to be the factors of the expression $x^2 - y^2$.

To prove that the two factors are as stated, we might multiply them together, and the result should be $x^2 - y^2$. The operation of multiplication appears below, and the description follows immediately:

$$\begin{array}{l} x + y \\ x - y \\ \hline x^2 + xy \\ \quad\;\; - xy - y^2 \\ \hline x^2 \qquad\quad - y^2 \\ \hline \end{array}$$

Place the two expressions $x + y$ and $x - y$ as shown. Multiply $x + y$ by x, then multiply $x + y$ by $-y$, and add the two results. The mental process is somewhat as follows: x times $x = x \times x$ or x^2; put down x^2 under the x. x times $y = x \times y$ or

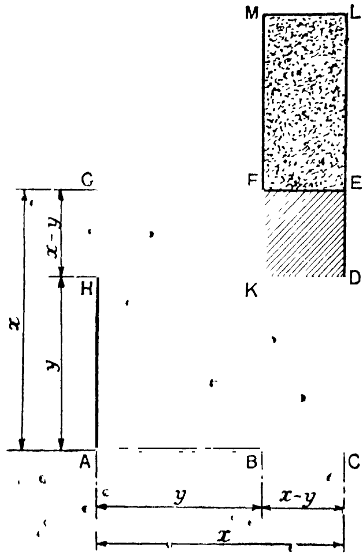

Fig. 18

xy; put down xy under the y. Now, multiply by $-y$; $-y$ times $x = -yx$ or $-xy$; put down $-xy$ under the $+xy$ already found. $-y$ times $y = -y^2$; put down $-y^2$ a little to the right. (Note: when the signs are the same, both + or both −, the product is + or positive; and when the signs are different, one − and one +, the product is − or negative.)

Now add together all the similar terms: there is only one x^2; put down x^2. Add $+xy$ to $-xy$; there is one of each, so the result is o or zero; leave the place blank as indicated. Finally, there is only one y^2, and it is negative, i.e. $-y^2$; write down $-y^2$. The result is thus $x^2 - y^2$.

The proof may also be shown graphically, as in fig. 18. ACEG is any square, and ABKH is a smaller square. The difference between the two is represented by a small shaded square KDEF, and two equal rectangles BCDK and HKFG.

$$\begin{aligned} \text{Let } x &= \text{AC, the side of the large square,} \\ \text{and } y &= \text{AB, the side of the small square.} \\ \text{Then BC} &= x - y, \\ \text{and GH} &= x - y. \end{aligned}$$

Suppose the upper rectangle HKFG be cut off and placed endways on the top of the shaded small square; it would appear as indicated by the stippled rectangle ELMF, so that

$$\begin{aligned} \text{Area of difference of squares} &= \text{rectangle BCLM.} \\ \text{But side BC} &= (x - y), \\ \text{and side CL} &= (x + y). \\ \therefore (x^2 - y^2) &= (x + y)(x - y). \end{aligned}$$

This important deduction may be applied to finding the area of a circular ring of any description. Thus,

$$\begin{aligned} \text{Let } d &= \text{the inside diameter of a plane circular ring,} \\ \text{and D} &= \text{the outside diameter of same plane circular ring.} \end{aligned}$$

$$\begin{aligned}
\text{Area of ring} &= \text{area of outer circle} - \text{area of inner circle} \\
&= \cdot 7854\,D^2 - \cdot 7854 d^2 \\
&= \cdot 7854\,(D^2 - d^2) \\
&= \cdot 7854\,(D + d)(D - d).
\end{aligned}$$

The two circles need not necessarily have the same centre, i.e. they need not be concentric.

Example 35.—A vertical boiling-kier 6 ft. 6 in. diameter is provided with a man-hole 3 ft. diameter. Find the weight of the top plate of the kier if it is made from metal weighing 18 lb. per square foot.

$$\begin{aligned}
\text{Area of plate} &= \text{area of upper surface} - \text{area of man-hole} \\
&= \text{area of 6 ft. 6 in.-diameter circle} - \text{area of 3 ft.-diameter circle} \\
&= \cdot 7854\,(D + d)(D - d) \\
&= \cdot 7854\,(6 \text{ ft. } 6 \text{ in.} + 3 \text{ ft.})(6 \text{ ft. } 6 \text{ in.} - 3 \text{ ft.}) \\
&= \cdot 7854 \times 9\tfrac{1}{2} \text{ ft.} \times 3\tfrac{1}{2} \text{ ft.} \\
&= \cdot 7854 \times 33 \cdot 25 \text{ sq. ft.} \\
&= 26 \cdot 115 \text{ sq. ft.} \\
\text{Weight of plate} &= 18 \text{ lb. per sq. ft.} \times 26 \cdot 115 \text{ sq. ft.} \\
&= 470 \text{ lb.}
\end{aligned}$$

VARIATION OF AREA OF CIRCLES.—The area of a circle $= \cdot 7854 d^2$. In this, $\cdot 7854$ is constant, therefore the area A varies directly as the square of the diameter, i.e. as d^2. d thus varies as the $\sqrt{A}$.

If the diameter is doubled, the area is increased fourfold. The diameter of a circle, for example, is 3 in.; the area $= \cdot 7854 \times 3^2 = 7 \cdot 07$ sq. ins. The area of a 6-in.-diameter circle is $\cdot 7854 \times 6^2 = 28 \cdot 28$ sq. in. The area of the large circle is, as stated,

4 times the area of the small circle, although their diameters are but 2 to 1. This fact can be deduced from the statement given above that the area of a circle varies as the square of the diameter, just as the areas of squares vary as the squares of their sides. Consider, for example, the diagrams in fig. 19; in each case there is a circle enclosed in a square, the side of which is equal to the diameter of the circle.

Upper diagram	square	$1 \times 1 = 1$	circle	$1^2 \times \cdot 7854$.
Middle ,,	,,	$2 \times 2 = 2^2$	,,	$2^2 \times \cdot 7854$.
Lower ,,	,,	$3 \times 3 = 3^2$	,,	$3^2 \times \cdot 7854$.

$$\frac{\text{Square}}{\text{Circle}} = \frac{1^2}{1^2 \times \cdot 7854} : \frac{2^2}{2^2 \times \cdot 7854} : \frac{3^2}{3^2 \times \cdot 7854}.$$

Multiplying each term by ·7854, we get

$$\frac{\cdot 7854 \text{ square}}{\text{circle}} : \frac{1^2 : 2^2 : 3^2}{1^2 : 2^2 : 3^2}$$

If, therefore, the diameters of the three diagrams in fig. 19 are in the proportion of 1 : 2 : 3, the corresponding areas will be in the proportion of 1 : 4 : 9, or $1^2 : 2^2 : 3^2$.

Again,

if area = 1, the diameter = $\sqrt{1}$,
,, = 2, ,, ,, = $\sqrt{2}$,
= 3, ,, ,, = $\sqrt{3}$,

and so on for all numbers.

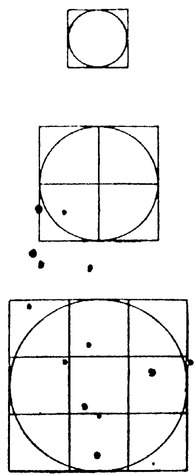

Fig 19

Example 36.—The main pipe leading from a starch-mixing apparatus has to serve four slasher sizing-machines, each with a supply-pipe of 2-in. diameter; assuming that each machine demands a regular supply of starch, what should be the diameters of the various sections of the main pipe?

Let 1 = area of machine supply-pipe E, fig. 20, whence the diameter = $\sqrt{1}$.

Section A of main pipe supplies 2 pipes E (1 machine).
,, A must have area 2, hence diameter = $\sqrt{2}$.
,, B supplies 4 pipes E (for 2 machines).
,, B must have area 4, hence diameter = $\sqrt{4}$.
,, C supplies 6 pipes E (for 3 machines).
,, C must have area 6, hence diameter = $\sqrt{6}$.
,, D supplies 8 pipes E (for all 4 machines).
,, D must have area 8, hence diameter = $\sqrt{8}$.

	E		A		B		C		D
Areas are thus in proportion,	1	:	2	:	4	:	6	:	8,
and diameters in proportion,	$\sqrt{1}$	:	$\sqrt{2}$	:	$\sqrt{4}$	:	$\sqrt{6}$	:	$\sqrt{8}$,
or	1	:	1·414	:	2	:	2·45	:	2·83.
If E = 2 in., the diameters are	2	:	1·414 × 2	:	2 × 2	:	2·45 × 2	:	2·83 × 2
=	2	:	2·828	:	4	:	4·9	:	5·66.
Correct values to nearest $\frac{1}{8}$ in. =	2 in.	:	$2\frac{7}{8}$ in.	:	4 in.	:	5 in.	:	$5\frac{5}{8}$ in.

Example 37.—The approximate diameter of a jute yarn, expressed as a fraction of an inch, is obtained by the formula: $d = \frac{\sqrt{c}}{120}$, where c is the count in pounds per spindle of 14,400 yd. Find the diameter of a 16-lb. yarn, and compare it with the diameter of a 9-lb. yarn.

$$\text{16-lb. yarn: } d = \frac{\sqrt{c}}{120} = \frac{\sqrt{16}}{120} = \frac{4}{120} = \frac{1}{30} \text{ in.}$$

$$\text{9-lb. yarn: } d = \frac{\sqrt{c}}{120} = \frac{\sqrt{9}}{120} = \frac{3}{120} = \frac{1}{40} \text{ in.}$$

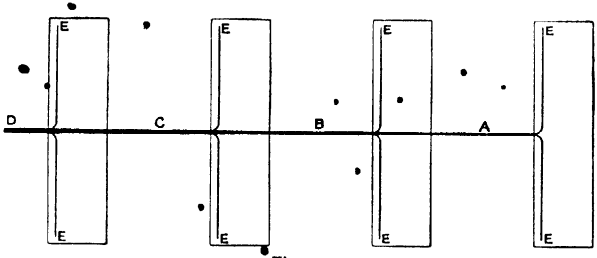

Fig. 20

Notice that the count of a yarn depends on its area, which, for comparative purposes, and as already demonstrated, may be taken as a circle. The area of a circle varies as the square of the diameter, or, in other words, the diameter varies as the square root of the area. In the case of jute yarn, since the area corresponds to the count, the diameters vary as the square roots of the counts. Thus, if 16-lb. yarn has a diameter of $\frac{1}{30}$ in., the diameter of 9-lb. yarn will be

$$\frac{1}{30} \times \frac{\sqrt{9 \text{ lb.}}}{\sqrt{16 \text{ lb.}}} = \frac{1}{30} \times \frac{3}{4} = \frac{1}{40} \text{ in.}$$

It should be particularly noticed that the above refers to jute; the diameters of cotton, woollen, spun silk, worsted, and linen yarns vary inversely as the square root of the count.

Exercises, with answers, on p. 114.

CHAPTER X

ARCS, CHORDS, SECTORS, AND SEGMENTS

ARCS.—An arc of a circle is the name given to any part of its circumference. Thus, in fig. 21, a circle is

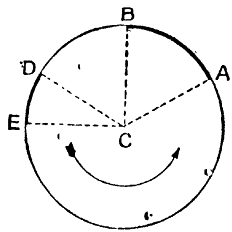

Fig. 21

illustrated with its centre at C. The heavy part in the circumference between A and B is an arc; similarly, DE is an arc, but a smaller one than AB; again, the two remaining parts of the circle in lighter lines are also arcs; hence, a circle may contain any number of arcs according to the size of the latter. If CA and CB are joined, the angle ACB is formed at the centre of the complete circle, and is termed the central angle. It is obvious that if this angle ACB is doubled, the length of the arc will also be doubled. The lengths of arcs of any particular circle are thus proportional to the central angle.

Suppose the radius CA rotates round the point C in the direction shown by the arrow, i.e. counter-clockwise. By the time it again reaches its present position it will have passed through an angle of 360°, or 4 right angles. The angular division known as a degree may be further divided into minutes and seconds in order to make more accurate measurements possible. Thus,

60″	or	60 seconds	=	1 minute.
60′	,,	60 minutes	=	1 degree.
360°	,,	360 degrees	=	1 complete circle = 4 right angles.

If the angle ACB, fig. 21, measures 72°, then the arc AB will be $\frac{72}{360}$ of the complete circumference; and if the circumference is 5 in., the arc will be $\frac{72}{360}$ of 5 in. = 1 in. long.

Let a = length of arc;
c = circumference of circle;
and D = the number of degrees.

$$\text{Then } a = \frac{D}{360}c = \frac{Dc}{360}.$$

It is most convenient, as already indicated, to measure circles by their diameters; since the circumference = πd, the value πd may be substituted for c in the above expression, giving

$$a = \frac{\pi d D}{360} = \frac{3 \cdot 1416}{360} d D = \cdot 00872 d D.$$

In the expression given there are thus three factors which may vary; these are a, d, and D; if any two are known, the third may readily be found by a suitable transposition of the formula. Thus,

$$a = \cdot 00872 d\mathrm{D} \quad \ldots\ldots\ldots\ldots (1)$$

$$d = \frac{a}{\cdot 00872\mathrm{D}} \quad \ldots\ldots\ldots\ldots (2)$$

$$\mathrm{D} = \frac{a}{\cdot 00872 d} \quad \ldots\ldots\ldots\ldots (3)$$

Example 38.—The diameter of a circle is 42 in., find the length of the arc which subtends an angle of 60° at the centre.

$$\begin{aligned} a &= \cdot 00872 d\mathrm{D} \\ &= \cdot 00872 \times 42 \times 60 \\ &= 21\cdot 9744 \text{ in.} \end{aligned}$$

Example 39.—The length of an arc of a circle 42 in. diameter is 21·9744 in., what angle does it subtend at the centre?

$$\begin{aligned} \mathrm{D} &= \frac{a}{\cdot 00872 d} \\ &= \frac{21\cdot 9744}{\cdot 00872 \times 42} \\ &= 60 \text{ degrees.} \end{aligned}$$

CHORDS.—If any two points on the circumference of a circle are joined by a straight line, this straight line is termed a chord. Thus, AB in fig. 22 is a chord of the circle ADBE. The centre of the circle is at C, and the radius CD bisects the chord AB, the two lines being perpendicular to each other. AB is also the chord of the arc ADB, as well as the chord of the arc AEB.

Since the radius CD is drawn at right angles to the chord AB, the point F is the centre of the chord, and the point D is the centre of the arc ADB. If CA and CB are joined, right-angled triangles CFA and CFB are formed. The distance FD, on the radius CD, is called the height of the arc ADB.

It has already been shown that in a right-angled triangle the square on the hypotenuse is equal to the

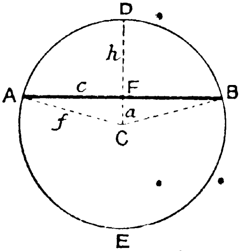

Fig. 22

sum of the squares on the other two sides. This fact may be used to solve many questions regarding chords. Thus, in the triangle AFC,

$$f^2 = c^2 + a^2,$$

f being the radius of the circle, c being half the chord, and a being the radius minus the height of the arc.

$$\text{Now, if } f^2 = c^2 + a^2,$$
$$f = \sqrt{c^2 + a^2}.$$

Similarly, by transposition, we have

$$c^2 = f^2 - a^2 \text{ or } c = \sqrt{f^2 - a^2},$$
$$\text{and } a^2 = f^2 - c^2 \text{ or } a = \sqrt{f^2 - c^2}.$$

Note that there are three variables involved, and if any two are given, the third may be found.

It is also possible to deduce the following rule, which should be specially used to find the diameter only when the chord and the height of the arc are given:

$$d = \frac{c^2 + h^2}{h}.$$

The expressions for h in terms of d and c, and for c in terms of h and d are rather unwieldy and seldom

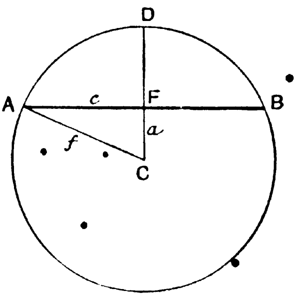

Fig. 23

used. If, however, the student desires practice, he can find them from the following data, which illustrates a method of finding d:

$$h(d - h) = c^2,$$
$$hd - h^2 = c^2,$$
$$hd = c^2 + h^2.$$
$$\therefore d = \frac{c^2 + h^2}{h}, \text{ as already stated.}$$

Example 40.—In a 20-in. circle, fig. 23, there is a chord drawn 4 in. from the centre; find the length of the chord.

The solution of this example appears in two ways:

$$\begin{aligned}
f^2 &= a^2 + c^2, \\
c^2 &= f^2 - a^2, \\
c &= \sqrt{f^2 - a^2}, \\
2c &= \text{chord} \\
&= 2\sqrt{f^2 - a^2} \\
&= 2\sqrt{10^2 - 4^2} \\
&= 2\sqrt{100 - 16} \\
&= 2\sqrt{84} \\
&= 2 \times 9 \cdot 165 \\
&= 18 \cdot 33 \text{ in.}
\end{aligned}$$

$$\begin{aligned}
c^2 &= h(d - h) \\
&= (10 - 4)(2 \times 10 - 6) \\
&= 6 \times 14 \\
&= 84, \\
c &= \sqrt{84}. \\
\therefore 2c &= 2\sqrt{84} \\
&= 2 \times 9 \cdot 165 \\
&= 18 \cdot 33 \text{ in.}
\end{aligned}$$

Example 41.—The chord of an arc is 40 ft., and its length 13 ft. Find the diameter of the circle.

$$\begin{aligned}
d &= \frac{c^2 + h^2}{h} \\
&= \frac{(\frac{40}{2})^2 + 13^2}{13} \\
&= \frac{400 + 169}{13} \\
&= \frac{569}{13} = 43 \cdot 77 \text{ ft.}
\end{aligned}$$

SECTORS.—A sector of a circle is that part which is bounded by an arc and 2 radii drawn from its ends to the centre, as shown in fig. 24, where ACB is a sector of the circle AEB.

The angle ACB is termed the sector angle, and in any circle the area of a sector is proportional to the sector angle. If the angle is 360°, the sector becomes a circle, and its area $= \frac{\pi}{4}d^2$, or πr^2. If the sector angle be D degrees, then

$$\text{Area A of sector} = \frac{D}{360} \times \frac{\pi}{4} d^2 \text{ or } \frac{D}{360}\pi r^2$$

$$= \frac{\cdot 7854}{360} Dd^2 \text{ or } \cdot 00872 Dr^2.$$

$$A = \cdot 00218 Dd^2.$$

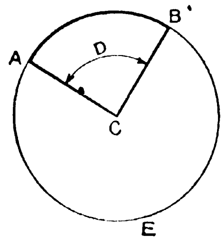

Fig. 24.

Example 42.—In a 20-in.-diameter circle, find the area of a sector the arc of which subtends an angle of 80° at the centre.

$$\text{Area A of sector} = \cdot 00218 Dd^2$$
$$= \cdot 00218 \times 80 \times 20^2$$
$$= \cdot 00218 \times 80 \times 20 \times 20.$$
$$A = 69 \cdot 76 \text{ sq. in.}$$

It is not always convenient to measure the sector angle; when this is the case, the arc may be measured instead. The formula then becomes:

$$\text{Area A of sector} = \frac{\text{arc}}{2} \times \text{radius}.$$

Example 43.—Find the area of a sector of a 20-in.-diameter circle, the arc of which is 40 in. in length.

$$\text{Area A of sector} = \tfrac{1}{2} \text{ arc} \times \text{radius}.$$
$$= \frac{40}{2} \times 10.$$
$$A = 20 \times 10 = 200 \text{ sq. in.}$$

This rule is based on the law for the area of a circle. If a sector angle be made near enough to 360°, the sector approaches more and more closely to a complete circle. When the sector angle reaches 360°, the figure is a circle. Now,

$$\begin{aligned} \text{Area } A \text{ of circle} &= \pi r^2 \\ &= \pi r \times r \\ &= \tfrac{1}{2} \text{ circumference} \times \text{radius.} \end{aligned}$$

In the case of a sector less than a circle, the circumference in the above is replaced by the value of the arc.

$\therefore$ Area A of sector $=$
$\tfrac{1}{2}$ arc $\times$ radius.

SEGMENTS.—A segment of a circle is that part bounded by a chord and its arc.

Thus, in fig. 25, ABDE is a circle with centre C. ABD is an arc with AD the corresponding chord. The shaded portion is a segment of the circle, while the unshaded portion is another segment.

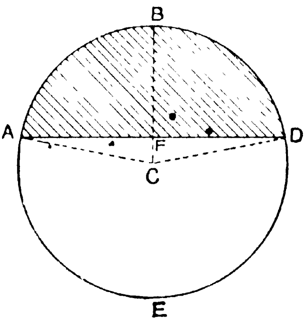

Fig. 25

If the radius CFB cuts AD at right angles, the point F is the centre of the chord AD, and the point B is the centre of the arc ABD.

Note that ABDCA is a sector of the circle, and that AFDC is an isosceles triangle of which the two sides CA and CD are radii of the circle, and the third side is the chord AD of the arc ABD.

$$\begin{aligned} \text{Area } A \text{ of segment ABD} &= \text{area of sector ABDC} \\ &\quad - \text{area of triangle ACD.} \end{aligned}$$

The area of the sector is found by the rules just discussed, viz. half the product of the arc and the radius, or $\cdot 00218 D d^2$. The area of the triangle may be found by any of the methods indicated in Chapter VI, or by any of the trigonometrical methods which are explained in a later chapter.

Example 44.—Find the area of the segment, the chord of which is 20 ins. long, the height 5 in., and the arc 26 in. long.

Referring to Example 41, we see that

$$d = \frac{c^2 + h^2}{h} \text{ (where } c = \tfrac{1}{2} \text{ chord)}$$

$$= \frac{10^2 + 5^2}{5}$$

$$= \frac{100 + 25}{5} = \frac{125}{5} = 25 \text{ in. diameter.}$$

If $d = 25$ in., $r = 12\frac{1}{2}$ in.

$$\therefore \text{ Area A of sector} = \tfrac{1}{2} \text{ arc} \times \text{radius}$$
$$= \tfrac{1}{2} \times 26 \times 12\tfrac{1}{2}$$
$$= 162 \cdot 5 \text{ sq. in.}$$

$$\text{Area of triangle} = \tfrac{1}{2} \text{ base} \times \text{altitude}$$
$$= \tfrac{1}{2} \text{ chord} \times (\text{radius} - \text{height})$$
$$= \tfrac{1}{2} \times 20 \times (12\tfrac{1}{2} - 5)$$
$$= 10 \times 7\tfrac{1}{2} = 75 \text{ sq. in.}$$

$$\text{Area of segment} = \text{area of sector} - \text{area of triangle}$$
$$= 162 \cdot 5 - 75$$
$$= 87 \cdot 5 \text{ sq. in.}$$

Exercises, with answers, on p. 116.

CHAPTER XI

GEAR WHEELS

Gear Wheels.—It may be an advantage at this stage to refer briefly to the relations between the circumferences and diameters of toothed wheels, so extensively used in textile machinery. Fig. 26 is a drawing of a small spur pinion containing 18 teeth.

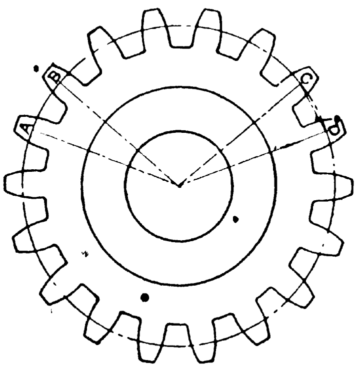

Fig 26

These teeth are situated at regular intervals on a circle known as the "pitch circle", and indicated by the outer circle which passes through every tooth. The distance between any point on the pitch circle of one tooth and the corresponding point on a neighbouring tooth is called the "pitch". Each pitch is obviously a certain definite measurement, and all wheels which are to gear together, or to form th same unbroken straight train, must have the sam pitch. In any machine wheel the pitch is determine

by two considerations: (*a*) the space available, and (*b*) the strength of tooth required. It is not proposed to deal with the strength question in this work, but merely to discuss the relations between pitch and diameter.

The diameter of the pitch circle is called the pitch diameter, and the pitch itself may be measured in at least three different ways.

1. The number of teeth in the rim may bear a certain relation to the pitch diameter of the wheel. Thus, in certain trains of gear, each wheel has 6 teeth in the rim for each inch in the pitch diameter, i.e. 6 teeth in 3·1416 in. of the pitch circle. For example:

$$\frac{18 \text{ teeth}}{3 \text{ in. pitch diameter}} = 6 \text{ teeth per inch of pitch diam.}$$

The number 6 expresses the ratio between the number of teeth in the wheel and the pitch diameter in inches. This ratio is briefly termed No. 6, and such a wheel is described as a spur pinion of 18 teeth, No. 6 pitch. This kind of pitch is termed *Diametral Pitch.* In general:

Let D = diameter of wheel in terms of pitch,
p_d = diametral pitch,
and N = number of teeth in the wheel.

Then $N = D \times p_d$,

$$D = \frac{N}{p_d},$$

and $p_d = \dfrac{N}{D}$.

2. Another common method of expressing the pitch is to state the actual distance in inches, or in a fraction of an inch, between corresponding points on two

adjacent teeth, measured along the pitch circle. For example, in the 18-tooth pinion illustrated in fig. 26, suppose the pitch circle is 18 in. in circumference, the distance between each pair of teeth, or the pitch of the teeth, will be 1 in. This kind of pitch is called *Circular Pitch* (see the arc AB in fig. 26).

In general:

Let D = the diameter of the pitch circle,
p_c = the circular pitch,
and N = the number of teeth in the wheel.

$$\text{Then } N = \frac{\pi D}{p_c},$$

$$D = \frac{N \times p_c}{\pi},$$

$$\text{and } p_c = \frac{\pi D}{N}.$$

3. When making complete patterns of gear wheels, it is not always easy to mark out the pitch along the pitch circle, and successive trials, with the necessary adjustments, are made until the correct division is obtained. Note that if any instrument of the divider type is used, the measurement marked off will be the straight line between the centres of two adjoining teeth. This straight-line measurement is really a chord of the pitch circle, such as chord CD in fig. 26. This kind of pitch is termed the *Chordal Pitch*.

The chordal pitch is not commonly used to express the relation between the pitch and the diameter; the calculation involved is rather cumbersome, and further discussion would be out of place in an elementary study of mathematical principles.

Example 41.—Two gear wheels (fig. 27) which mesh directly with one another contain 50 and 25 teeth respectively. The 25-tooth wheel is keyed to a

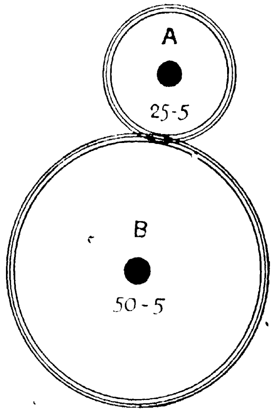

FIG. 27

loom crank-shaft, and the 50-tooth wheel is keyed to the bottom or low shaft (called wyper-shaft or cam-shaft). If the pitch is No. 5, find the exact distance between the centres of the two shafts.

Distance D

$= \frac{1}{2}$ diam. of wheel A + $\frac{1}{2}$ diam. of wheel B

$= \frac{1}{2}$ (diam. of wheel A + diam. of wheel B)

$= \frac{1}{2}$ (25 teeth 5 pitch + 50 teeth 5 pitch)

$= \frac{1}{2}$ (75 teeth 5 pitch)

$= \frac{1}{2} \times \frac{75}{5}$

$= 7\frac{1}{2}$ in. between the centres.

Example 46.—A 72-tooth wheel on a factory main-shaft is to have the teeth set at 2-in.-circular pitch. Find the pitch diameter, and calculate the mean speed of the wheel rim in feet per minute if the shaft makes 200 revolutions per minute.

$$D = \frac{N \times p_c}{\pi}$$

$$= \frac{72 \times 2}{3 \cdot 14}$$

$= 45 \cdot 86$ in. or $3 \cdot 82$ ft. pitch diameter.

Mean speed $=$ mean circumference in feet $\times$ r.p.m.

$$= \frac{72 \times 2}{12} \times 200$$

$= 12 \times 200 = 2400$ ft. per minute.

Exercises, with answers, on p. 118.

CHAPTER XII

RECTANGULAR SOLIDS, RELATIVE DENSITY, AND SPECIFIC GRAVITY

Rectangular Solid.—A rectangular solid is a body bounded by six rectangular faces, each pair of opposite faces being parallel and equal in area. Two examples of rectangular solids appear in fig. 28; the

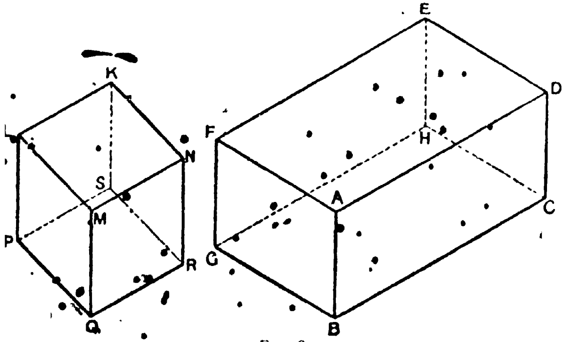

Fig. 28

small one on the left is a special form, known as a cube, each of its six faces being a square.

The rectangular solid on the right has six rectangular faces, each pair of opposite faces, ABCD and EFGH, ABGF and DCHE, ADEF and BCHG, being

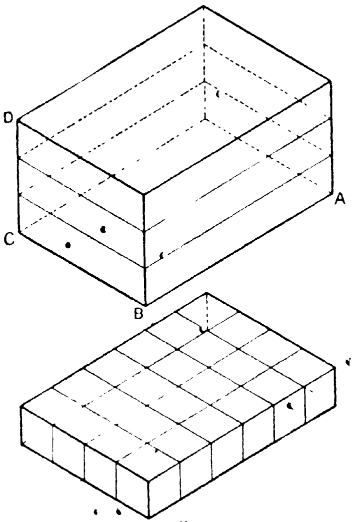

Fig. 29

equal and parallel rectangles. The length, breadth, and height of this particular solid are all different; when these three factors are alike in measurement, the rectangular solid becomes a cube, as already defined.

The amount of space occupied by a solid is called the volume, and volumes are measured in cubic inches, cubic feet, or other suitable cubic unit. In the above specified cases there are

$$12^3 = 12 \times 12 \times 12 = 1728 \text{ cu. in. in 1 cu. ft.}$$
$$3^3 = 3 \times 3 \times 3 = 27 \text{ cu. ft. in 1 cu. yd.}$$

Let ABCD, the upper diagram in fig. 29, be a rectangular solid, the measurements of which are: length AB = 6 units, breadth BC = 4 units, and height CD = 3 units. The solid may be divided into three slices, as indicated, one of the slices, or

rather a similar one, appearing in the lower diagram. Each of the slices may be divided into four long strips, each strip containing 6 units of 1 unit side. Suppose the unit to be 1 in., then each little cube is 1 cu. in. In each slice there will be four strips of 6 cu. in. = 24 cu. in., and in the three slices of the complete solid there will be $3 \times 24 = 72$ cu. in. The volume of the solid in the upper diagram in fig. 29 is therefore

$$6 \times 4 \times 3 = 72 \text{ cu. in.}$$

In general terms, the volume V of any rectangular solid is equal to the product of its length, breadth, and height; that is to say,

$$V = l \times b \times h,$$
$$\text{or } V = lbh.$$

From this general formula, which contains four variables, any unknown value of a term may be found when the remaining three are known.

$$V = lbh$$
$$l = \frac{V}{bh}$$
$$b = \frac{V}{lh}$$
$$\text{and } h = \frac{V}{lb}$$

Note that bh, lh, and lb are the areas of the various faces.

Example 47.—A bale of Indian cotton is 48 in. long, 20 in. wide, and 18 in. thick; find its volume in cubic feet.

$$\begin{aligned} V &= l \times b \times h \\ &= (48 \times 20 \times 18) \text{ cu. in.} \\ &= \frac{48 \times 20 \times 18}{1728} \text{ cu. ft.} \\ &= 10 \text{ cu. ft.} \end{aligned}$$

Example 48.—How many bales of the above cotton could be stored in a warehouse 120 ft. long, 80 ft. wide, and 40 ft. available height. (No reduction for cartways, &c.)

$$\text{Volume V} = l \times b \times h$$
$$= (120 \times 80 \times 40) \text{ cu. ft.}$$
$$= 384{,}000 \text{ cu. ft.}$$
$$\text{No. of bales} = \frac{\text{total volume of warehouse}}{\text{volume of 1 bale}}$$
$$= \frac{384{,}000 \text{ cu. ft.}}{10 \text{ cu. ft.}} = 38{,}400 \text{ bales.}$$

RELATIVE DENSITY.—It is often required to know how the weight of a given volume of one material compares with the weight of an equal volume of some other material. The weight of a substance measured in this way, i.e. weight per unit volume, is known as the density of the substance.

Example 49.—Find the relative density of a bale of Indian cotton, 10 cu. ft. in volume, and weighing 396 lb., and a Brazilian bale measuring 49 in. × 20 in. × 18 in., and weighing 220 lb.

Density of Indian bale

$$= \frac{396 \text{ lb.}}{10 \text{ cu. ft.}} = 39{\cdot}6 \text{ lb. per cu. ft.}$$

Density of Brazilian bale

$$= \frac{220 \text{ lb.}}{(49 \times 20 \times 18) \text{ cu. in.}}$$
$$= \frac{220 \text{ lb.}}{\frac{49 \times 20 \times 18}{1728} \text{ cu. ft.}}$$
$$= \frac{220 \text{ lb.}}{10{\cdot}21 \text{ cu. ft.}}$$
$$= 21{\cdot}55 \text{ lb. per cubic foot.}$$

Relative Density

$$= \frac{\text{Indian}}{\text{Brazilian}} = \frac{39 \cdot 6}{21 \cdot 55} = \frac{1 \cdot 84}{1}.$$

In other words, baled Indian cotton is, bulk for bulk, 1·84 times heavier than baled Brazilian cotton. If Brazilian cotton is taken as the standard of comparison, then

Relative Density

$$= \frac{\text{Brazilian}}{\text{Indian}} = \frac{21 \cdot 55}{39 \cdot 6} = \cdot 547 = \frac{1}{1 \cdot 84};$$

that is to say, baled Brazilian cotton is only ·547 times as heavy as baled Indian cotton.

SPECIFIC GRAVITY.—In Example 49, baled Indian and baled Brazilian cotton were in turn used as standards of comparison, and the relative densities of each with respect to the other were found. It is, however, much more convenient to have one invariable standard of comparison, and this standard of comparison is pure water. The density of water is regarded as unity, i.e. its numerical value is taken to be 1, and all other densities thus become:

(*a*) fractions of 1 if the substances are lighter than water, bulk for bulk, or,

(*b*) multiples of 1 if the substances are heavier than water, bulk for bulk.

When water is used as the standard of comparison, the ratio is given a special name, viz. *Specific Gravity*; that is to say, the number given to the material expresses the ratio of the weight of a given volume of the substance to the weight of an equal volume of water, when the weight of the latter is represented by 1. (Water is used as the standard

for all solids and liquids, while hydrogen is the standard for all gases.)

If it is known, for example, that the specific gravity of aluminium is 2·58, then it is understood that 1 cu. ft. of aluminium weighs 2·58 times as much as 1 cu. ft. of water.

The following particulars concerning the weight and volume of water should be committed to memory, as they are very important:

1 cu. ft. of water = 1000 oz. = $62\frac{1}{2}$ lb.
1 gall. of water = $277\frac{1}{2}$ cu. in. = 10 lb.
1 cu. ft. of water = $6\frac{1}{4}$ gall.

The above values are only approximate ones, but they are sufficiently accurate for all practical purposes. (If very accurate results are required, then 1 cu. ft. of water may be taken at 996·46 oz.; 1 gall. at 277·46 cu. in.; and 1 cu. ft. at 6·228 gall.)

Example 50.—The guide-bars of a packing-press measure 11 ft. 3 in. long by 6 in. wide and $1\frac{1}{4}$ in. thick. Find their weight if the specific gravity of steel is 7·8.

$$\begin{aligned} V &= l \times b \times h \\ &= 11 \text{ ft. } 3 \text{ in.} \times 6 \text{ in.} \times 1\tfrac{1}{4} \text{ in.} \\ &= \frac{135 \times 6 \times 1\cdot25}{1728} \text{ cu. ft.} \\ &= 0\cdot586 \text{ cu. ft.} \end{aligned}$$

$$\begin{aligned} 1 \text{ cu. ft. of water} &= 1000 \text{ oz.} \\ \cdot586 \text{ cu. ft.} &= \frac{\cdot586 \times 1000}{16 \text{ oz. per lb.}} \text{ lb.} \\ &= 36\cdot6 \text{ lb.} \end{aligned}$$

Specific gravity of steel is 7·8, therefore

$$\begin{aligned} \text{Weight of bar} &= 7\cdot8 \times 36\cdot6 \text{ lb.} \\ &= 285\cdot48 \text{ lb.} \end{aligned}$$

Area of Surfaces.—If a rectangular solid has length l, breadth b, and height h, the whole outer surface of the solid will be

$$\begin{aligned} &2(l \times h) + 2(l \times b) + 2(h \times b) \\ = \; &2(lh + lb + hb) \text{ sq. units.} \end{aligned}$$

If a cube has a side s, the area of the outer surface will be

$$\begin{aligned} &2(s \times s) + 2(s \times s) + 2(s \times s) \\ = \; &6(s \times s) = 6s^2 \text{ sq. units.} \end{aligned}$$

Example 51. — An acid-tank in a bleachfield measures 10 ft. long by 6 ft. wide by 4 ft. deep. Find the cost of lining it with sheet lead at 1*s.* per square foot.

Area of tank

$$= (l \times b) + 2(l \times h) + 2(h \times b)$$

N.B.—There are 4 sides and bottom (no top).

$$\begin{aligned} &= lb + 2(lh + hb) \\ &= (10 \times 6) + 2\{(10 \times 4) + (4 \times 6)\} \\ &= 60 + 2(40 + 24) \\ &= 60 + 80 + 48 \\ &= 188 \text{ sq. ft.} \end{aligned}$$

Cost = 188 sq. ft. × 1*s.* per square foot
= 188*s.* or £9, 8*s.*

The thickness of the wood has been ignored.
Exercises, with answers, on p. 119.

CHAPTER XIII

PRISMS AND CYLINDERS

PRISMS.—A prism is a solid bounded by plane faces, of which two, called ends, are parallel, equal in area, and exactly similar, while the other faces are parallelograms. Fig. 30 shows two forms of prism; in each

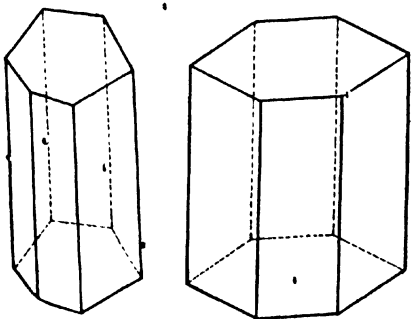

Fig 30

the ends are equal in area, parallel to each other, and of the same size and shape. The figure marked A is called an oblique prism, because the sides are not perpendicular to the lower end or base; the other figure, marked B, is termed a right prism, because its lateral edges are at right angles to the base.

The examples shown are hexagonal prisms, because the two ends are hexagons; the base of a prism may, however, have any shape, provided that the area of the end be bounded by straight lines.

AREA OF SURFACE OF PRISM.—The area of the surface of the prism is the area of the two ends added

to the area of the lateral sides. In the majority of cases which occur in practice, the sides are rectangles, so that the finding of the area should present little difficulty.

There is, however, a short method of finding the area of the lateral sides. It will be evident that if a piece of paper be cut to the size necessary to cover exactly the lateral sides of a right prism, and the paper then removed and laid flat, that it will form a rectangle in which the length equals the perimeter of the prism, and the breadth equals the height of the prism. Then,

Area of surface of prism = area of 2 ends + (perimeter of base × height of prism).

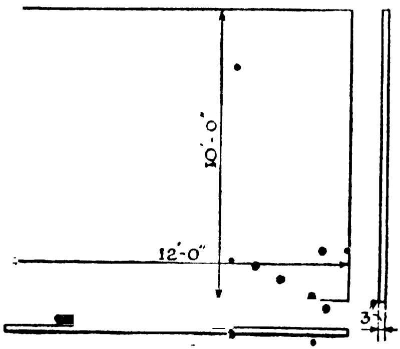

Fig 31

Example 52.—A plain rectangular warehouse door, 10 ft. high by 12 ft. wide by 3 in. thick, is to be fireproofed by covering it with sheets of tinned steel. Find the area of the sheet required, neglecting waste. The dimensions are shown in fig. 31.

Area of surface

$= \text{Area of 2 ends} + (\text{perimeter} \times \text{height})$

$= 2(12 \text{ ft.} \times 3 \text{ in.}) + \{(12 \text{ ft.} + 3 \text{ in.} + 12 \text{ ft.} + 3 \text{ in.}) \times 10 \text{ ft.}\}$

$= (2 \times 12 \times \frac{1}{4}) + (24\frac{1}{2} \times 10)$

$= (6 + 245) \text{ sq. ft.}$

$= 251 \text{ sq. ft.}$

VOLUME OF A PRISM.—The rectangular solid illustrated in fig. 29, p. 70, is evidently only a special form of prism. It was shown that its volume was obtained from the formula

$$V = l \times b \times h;$$

but $(l \times b) = $ area of base.

$$\therefore V = \text{area of base} \times \text{height}.$$

The same reasoning applies to any prism. The volume of a prism is, therefore,

$$V = \text{area of base} \times \text{height}.$$

Example 53.—Find the weight of a hexagonal bar of iron 12 ft. long by 1 in. side, it being given that 1 cu. ft. of iron weighs 484 lb.

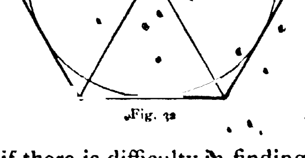

Fig. 32

NOTE.—A regular hexagon is made up of 6 equilateral triangles (see fig. 32). The area of an equilateral triangle may be found in the usual way if its altitude be known, but if there is difficulty in finding the altitude, use may be made of a special rule, based on the trigonometrical ratios of its angles (discussed in Chapter VII, Part II); in this case, if the length of side be l units,

$$\text{Area of equilateral triangle} = \frac{l^2\sqrt{3}}{4}.$$

$$\therefore \text{Area of hexagon} = 6 \times \frac{l^2\sqrt{3}}{4}$$

$$= \frac{6 \times l^2 \times 1 \cdot 732}{4} = 2 \cdot 598\, l^2,$$

so that in the example under discussion we have

$$\begin{aligned}
\text{Volume V} &= 2 \cdot 598\, l^2 \times \text{L (where L} = 12 \text{ ft.)} \\
&= 2 \cdot 598 \times 2^2 \times 12 \text{ ft.} \\
&= (2 \cdot 598 \times 4 \times 144) \text{ cu. in.} \\
&= \frac{2 \cdot 598 \times 4 \times 144}{1728} \text{ cu. ft.} \\
&= 0 \cdot 866 \text{ cu. ft.} \\
\text{Weight of bar} &= \cdot 866 \times 484 \text{ lb.} \\
&= 419 \cdot 144 \text{ lb.}
\end{aligned}$$

CYLINDER.—A cylinder may be roughly described as a special form of prism with circular ends. A right circular cylinder is more correctly defined as the solid described by one complete revolution of a rectangle, one side of which acts as the fulcrum. Thus, in fig. 33, ABCD is a rectangle of which the side AB is the line about which the rectangle may be assumed to revolve in order to sweep out in space a volume of the shape known as a right cylinder. It is one of the commonest shapes adopted in textile machinery, and indeed in all mechanical work, and its mathematical properties are therefore of extreme importance.

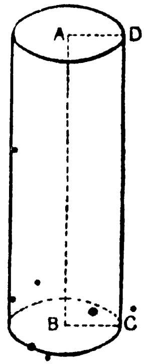

Fig. 33

The complete surface of the cylinder consists of the

curved surface swept out by the side CD, fig 33, and of the two circular surfaces or ends swept out by the sides BC and AD.

The dimensions of a cylinder are generally expressed by stating the diameter and the length or height, and practical mathematical laws should therefore be stated in terms of these dimensions.

SURFACE OF A CYLINDER.—The area of the curved surface of a cylinder is that traced out by a line, equal to the height or length, moving through a distance equal to the circumference of the end, so that

$$\begin{aligned} \text{Area of curved surface} &= \text{circumference of base} \times \text{height or length} \\ &= \pi d \times l, \\ \text{or surface S} &= \pi dl. \end{aligned}$$

The whole surface area will be equal to the above added to the two circular ends, or, stated symbolically,

$$\begin{aligned} A &= \pi dl + 2 \times \frac{\pi}{4} \times d^2 \\ &= \pi d\left(l + \frac{d}{2}\right) \\ &= \pi d(l + r), \text{ where } r = \text{radius}. \end{aligned}$$

The volume of the cylinder will be

$$\begin{aligned} V &= \text{area of base or section} \times \text{length or height} \\ &= \frac{\pi}{4} d^2 \times l \\ &= \frac{\pi}{4} d^2 l, \text{ or } \cdot 7854 d^2 l. \end{aligned}$$

If preferable, the equation may be stated in terms of the radius. Thus,

$$V = \pi r^2 l.$$

Example 54.—The colour-drum of a calico-printing machine is 6 ft. diameter, and runs at 5 r.p.m., printing cloth 60 in. wide. Find the number of square yards of cloth capable of being printed in one hour.

Curved surface of drum, or $S = \pi dl$

$= 3 \cdot 1416 \times 6 \text{ ft.} \times 60 \text{ in.}$

$= (3 \cdot 1416 \times 6 \times 5) \text{ sq. ft.}$

$= 94 \cdot 248 \text{ sq. ft. in 1 rev.}$

Area printed in 1 min. $= (94 \cdot 248 \times 5 \text{ revs.}) \text{ sq. ft.}$

" " " 1 hour $= (94 \cdot 248 \times 5 \times 60) \text{ sq. ft.}$

$$= \frac{94 \cdot 248 \times 5 \times 60}{9} \text{ sq. yd.}$$

$= 3141 \cdot 6 \text{ sq. yd.}$

Example 55.—The starch or sowbox roller of a cylinder dressing-, slashing-, or sizing-machine is 18 in. diameter and 66 in. long; find the area in square feet of the copper sheet required to cover it completely.

Whole area $A = \pi d(l + r)$

$= \{3 \cdot 1416 \times 18(66 + \frac{18}{2})\} \text{ sq. in.}$

$= 3 \cdot 1416 \times 18(66 + 9) \text{ sq. in.}$

$$= \frac{3 \cdot 1416 \times 18 \times 75}{144} \text{ sq. ft.}$$

$= 29 \cdot 45 \text{ sq. ft.}$

Example 56.—A roller weight used in a drawing-frame is $3\frac{1}{2}$ in. diameter and 8 in. long; find its weight if 1 cu. in. of cast iron weigh $\cdot 263$ lb.

$$V = \frac{\pi d^2 l}{4}$$

$= (\cdot 7854 \times 3\frac{1}{2} \times 3\frac{1}{2} \times 8) \text{ cu. in.}$

$= 76 \cdot 97 \text{ cu. in.}$

Weight $= 76 \cdot 97 \text{ cu. in.} \times \cdot 263 \text{ lb. per cubic inch.}$

$= 20 \cdot 24 \text{ lb., say } 20\frac{1}{4} \text{ lb.}$

HOLLOW CYLINDERS.—In practice, it is customary, whenever feasible, to use hollow cylindrical objects in preference to solid cylinders. In such a case, the volume of the hollow cylinder only is required, and suitable rules for this calculation can be deduced from the general statement given above.

Example 57.—Fig. 34 illustrates an extensively-used hollow cylinder, that of a delivery pressing-roller

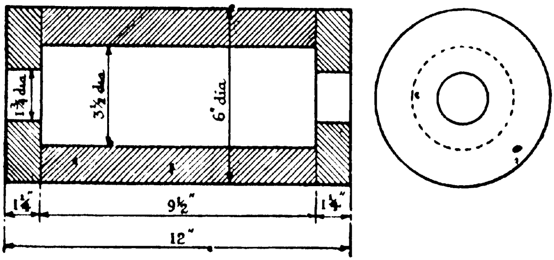

Fig. 34

suitable for a flax or jute drawing-frame. The roller consists of three hollow cylinders joined together—two end-pieces and one centre-piece.

$$\left.\begin{array}{l}\text{Volume of centre-}\\\text{piece}\end{array}\right\} = \begin{array}{l}\text{volume of solid cylinder} -\\\text{volume of hole}\end{array}$$

$$= \frac{\pi}{4}D^2l - \frac{\pi}{4}d^2l$$

$$= \frac{\pi}{4}l(D^2 - d^2)$$

$$= \frac{\pi}{4}l(D + d)(D - d)$$

$$= \frac{\pi}{4} \times 9\tfrac{1}{2}(6 + 3\tfrac{1}{2})(6 - 3\tfrac{1}{2})$$

$$= \cdot 7854 \times 9\tfrac{1}{2} \times 9\tfrac{1}{2} \times 2\tfrac{1}{2}$$

$$= 177\cdot 21 \text{ cu. in.}$$

$$\left.\begin{array}{l}\text{Volume of 2 end-}\\ \quad\text{pieces}\end{array}\right\} = 2\left\{\frac{\pi}{4}l(D^2 - d_1^2)\right\}$$

$$= 2\{\cdot 7854 \times 1\tfrac{1}{4}(D + d_1)(D - d_1)\}$$

$$= 2 \times \cdot 7854 \times 1\tfrac{1}{4} \times (6 + 1\tfrac{3}{4})(6 - 1\tfrac{3}{4})$$

$$= 2 \times \cdot 7854 \times 1\tfrac{1}{4} \times 7\tfrac{3}{4} \times 4\tfrac{1}{4}$$

$$= 64\cdot 67 \text{ cu. in.}$$

$$\left.\begin{array}{l}\text{Volume of whole}\\ \quad\text{roller}\end{array}\right\} = 177\cdot 21 + 64\cdot 67$$

$$= 241\cdot 88 \text{ cu. in.}$$

If the roller is made of cast iron weighing $\cdot 263$ lb. per cubic inch, then

$$\text{Weight of roller} = 241\cdot 88 \text{ cu. in.} \times \cdot 263 \text{ lb. per cu. in.}$$
$$= 63\cdot 61 \text{ lb.}$$

Example 58.—Find the volume occupied by rove yarn as represented by the shaded portion in fig. 35,

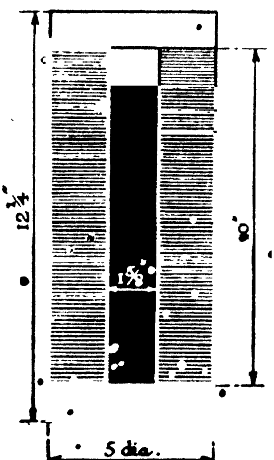

Fig. 35

and compare the volume thus found with that occupied by the block outline of the full bobbin.

$$\text{Block volume } V = \frac{\pi}{4} D^2 L$$
$$= \cdot 7854 \times 5 \times 5 \times 12\tfrac{1}{4}$$
$$= 240 \cdot 53 \text{ cu. in.}$$

$$\text{Volume } V_1 \text{ of yarn} = \frac{\pi}{4} l(D + d)(D - d)$$
$$= \cdot 7854 \times 10(5 + 1\tfrac{5}{8})(5 - 1\tfrac{5}{8})$$
$$= \cdot 7854 \times 10 \times 6\tfrac{5}{8} \times 3\tfrac{3}{8}$$
$$= 175 \cdot 61 \text{ cu. in.}$$

$$\frac{\text{Volume of rove}}{\text{Block vol. of bobbin}} = \frac{V_1}{V} = \frac{175 \cdot 61}{240 \cdot 53}.$$

$$\% \text{ vol. occupied by rove} = \frac{175 \cdot 61 \times 100}{240 \cdot 53} = 73\,\%.$$

Exercises, with answers, p. 120.

CHAPTER XIV

PYRAMID, CONE, AND SPHERE

The pyramid, cone, and sphere are three geometrical solids each of which has but a limited application to textile work. The proofs of the mathematical principles upon which their areas and volumes depend are comparatively difficult, and it is not proposed to discuss them at this stage. The paragraphs immediately following do not, therefore, include a full exposition of the principles involved, but may rather be regarded as a summary of the required definitions and rules, with a few worked-out examples to illustrate their application.

THE PYRAMID.—The name pyramid is applied to any solid which is bounded by plane surfaces, one of which, termed the base, may be any rectilinear figure, while the others are all triangles. The triangular faces have a common vertex, which must of necessity lie outside the plane of the base. The point may be near the base, or a distance removed from it, and fig. 36 illustrates one form of pyramid in which the distance of the point from the base is greater than the width of the base ABCDEF. The triangular sides are represented by ABV, BCV .... FAV, all of which taper to the point V, the common vertex.

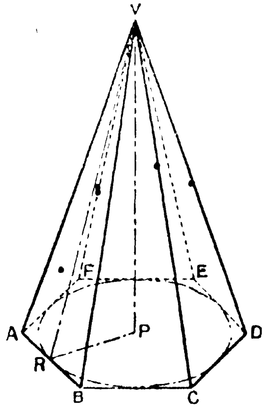

Fig 36

The pyramid illustrated is fully described as a right hexagonal prism. It has already been indicated why it may be termed a prism. It is a hexagonal prism because its base has 6 sides, and is therefore a hexagon; it is a right prism because the line VP, a perpendicular dropped from the vertex V to the base, meets the base at its centre P. Point P is the centre of the inscribed circle indicated; it is also the centre of the circumscribed circle which is not shown, but which would pass outside the hexagon and touch all points A, B, C, D, E, F. If the base had happened to be a rectangle, the point P would have coincided with the crossing or intersection of the diagonals.

The distance VP is called the *height* of the pyramid, while the distance VR is termed, for the sake of distinction, the *slant height*. The combined area of the triangular faces is the sum of the slant surfaces. The total surface area of the pyramid is, of course, the area of the slant surfaces + the area of the base.

VOLUME OF PYRAMID.—It may be proved that the volume of a pyramid is one-third of the volume of a prism which has the same base and the same perpendicular height.

If V = the volume,
h = the height,
and A = the area of the base, then

$$V = \frac{Ah}{3}.$$

With this formula as a starting-point, it will be a useful exercise for the student to deduce an expression for A in terms of V and h, and then one for h in terms of V and A.

SLANT SURFACE OF PYRAMID.—Since all the slant surfaces of the pyramid are triangles, the area of each of which is $\frac{1}{2}$ base × altitude, it is evident that

$$\text{Surface } S = \frac{ps}{2},$$

where S = the area of the slant surfaces,
p = the perimeter of the base,
and s = the slant height.

The above formula applies only to pyramids with regular bases, as this type is the only form of pyramid in which the slant heights of the various triangular faces are uniform.

WHOLE SURFACE OF PYRAMID.—As previously

indicated, the whole surface area of the pyramid is represented by

Area of slant surfaces + area of base.

The method of finding the area of the slant surfaces has just been discussed; the area of the base is obtained by applying the rules relating to rectilineal figures given in previous chapters.

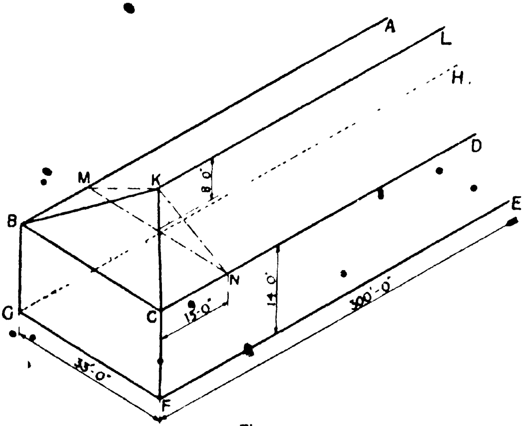

Fig. 37

Example 59.—Fig. 37 represents part of one bay of a weaving-shed, the inside dimensions being as indicated. Find the maximum number of persons which may be employed in a shed consisting of 6 similar bays, given that the Factory Acts require 250 cu. ft. of space per person.

An examination of the figure shows that each bay consists of a rectangular prism ABCDEFGH, 300 ft. long, 33 ft. broad, and 14 ft. high; a triangular prism DNMALK, 33 ft. wide, 8 ft. high, and

300 − (2 × 15) = 270 ft. long; and two halves (one at each end) of a rectangular pyramid KMNCB, each with a base 33 ft. by 15 ft., and a height of 8 ft.

Cubical contents of one bay

$$= \text{Rect. prism} + \text{triang. prism} + \text{rect. pyramid}$$

$$= lbh + Al + \frac{Ah}{3}$$

$$= (300 \times 33 \times 14) + \frac{ab}{2}l + \frac{lbh}{3}$$

$$= (300 \times 33 \times 14) + \left(\frac{8 \times 33}{2} \times 270\right) + \left(\frac{32 \times (2 \times 15) \times 8}{3}\right)$$

$$= 138{,}600 + 35640 + 2640$$

$$= 176{,}880 \text{ cu. ft.}$$

Cubical contents of 6 bays

$$= 176{,}880 \times 6 = 1{,}061{,}280 \text{ cu. ft.}$$

Maximum number of employees

$$= \frac{1{,}061{,}280}{250} = 4245 \text{ persons.}$$

NOTE.—The number of workers actually employed in a factory of such dimensions would be considerably less than the above, since the floor space for each operative would be less than 14 sq. ft.; actually,

$$\frac{33 \times 300 \times 6}{4245} = 13 \cdot 99 \text{ sq. ft.}$$

The calculation shows, however, the generous proportions of modern mills and factories compared with the requirements of the Factory Acts.

CONE.—Just as the circle may be regarded as a special form of polygon, so also may the cone be regarded as a special form of pyramid. If the length of the sides of the hexagonal base of the pyramid in

fig. 36, p. 85, be decreased, and the number of sides increased, it will be seen that the base will approach more and more closely in shape to a circle. When the length of each side is reduced to a point, that is, infinitely small, and the number of such sides becomes infinitely large, the base actually becomes a circle, and the pyramid itself is changed to a cone.

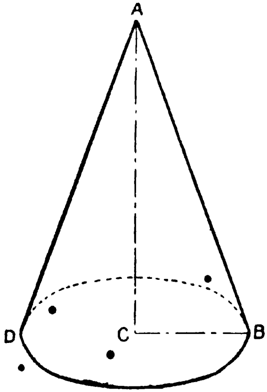

Fig. 38

A right circular cone may be defined as the solid swept out by the revolution of a right-angled triangle, one of the sides containing the right angle acting as a fulcrum. Fig. 38 represents such a cone, swept out by a complete revolution of the right-angled triangle ACB, $\hat{ACB}$ being a right angle. AC is the axis or height of the cone; AB is the slant height; while $\hat{BAD}$, the angle at the apex, is termed the vertical angle.

The whole surface area of the cone consists of the

circular base due to the rotation of the side CB of the triangle, and the curved surface resulting from the rotation of the hypotenuse BA of the triangle. It will be understood that the cone bears the same relation to the cylinder as the pyramid does to the prism. The expressions for finding the volume of the cone, and for obtaining the surface areas, will consequently correspond.

VOLUME OF CONE.—If V is the volume, A the area of the base, and h the height, it may be proved that

$$V = \frac{Ah}{3}.$$

In the case of a cone, the base is always a circle of diameter D. The above formula may, therefore, be more completely expressed by substituting $\frac{\pi}{4}D^2$ for A, when we have

$$V = \frac{\frac{\pi}{4}D^2h}{3}$$
$$= \frac{\cdot 7854D^2h}{3} = \cdot 2618D^2h.$$

CURVED SURFACE OF CONE. — Note that in the case of a regular pyramid, the area of the slant surface S, as shown on p. 86, is $\frac{ps}{2}$, p being the perimeter of the base, and s the slant height. In the case of the cone, the perimeter of the base is evidently the circumference of a circle of diameter D.

$$\therefore S = \frac{\pi Ds}{2} = \frac{3\cdot 1416Ds}{2} = 1\cdot 5708Ds,$$

or, in terms of radius $= 3\cdot 1416rs = \pi rs$.

If it is easier to measure the vertical height than the slant height, use may be made of the properties of the right-angled triangle to deduce a formula expressing S in terms of the base D and the vertical height h. Thus,

$$\begin{aligned} S &= 1 \cdot 5708 Ds \\ &= 1 \cdot 5708 D \sqrt{h^2 + \left(\frac{D}{2}\right)^2} \\ &= 1 \cdot 5708 \times 2r \times \sqrt{h^2 + r^2} \\ &= 3 \cdot 1416 \times r \sqrt{h^2 + r^2} \\ &= \pi r \sqrt{h^2 + r^2}. \end{aligned}$$

WHOLE SURFACE OF CONE.—The whole surface of the cone is evidently as under:

$$\begin{aligned} \text{Whole surface area} &= \text{area of base} + \text{area of curved surface} \\ &= \frac{\pi}{4} D^2 + 1 \cdot 5708 Ds \\ &= \pi r^2 + \pi rs \\ &= \pi r(r + s) \\ &= \pi r(r + \sqrt{h^2 + r^2}) \end{aligned}$$

Example 60.—Find the minimum length of canvas, 24 in. wide, required for the making of a bell tent of the dimensions shown in fig. 39.

$$\begin{aligned} \text{Area of tent} &= \text{curved surface of cylinder ABCD} \\ &\quad + \text{curved surface of cone EDC.} \\ A &= \pi dl + \pi r \sqrt{h^2 + r^2} \\ &= 2\pi rl + \pi r \sqrt{h^2 + r^2} \\ &= \pi r(2l + \sqrt{h^2 + r^2}) \\ &= 3 \cdot 1416 \times 5(2 \times 1 + \sqrt{9^2 + 5^2}) \\ &= 15 \cdot 708(2 + \sqrt{106}) \\ &= 15 \cdot 708 \times 12 \cdot 295 \\ &= 193 \cdot 13 \text{ sq. ft.} \end{aligned}$$

$$\text{Length of canvas} = \frac{\text{area}}{\text{width}}$$

$$= \frac{193 \cdot 13 \text{ sq. ft.}}{2 \text{ ft.}}$$

$$= 96 \cdot 565 \text{ ft.}$$

$$= 32 \cdot 188 \text{ yd.}$$

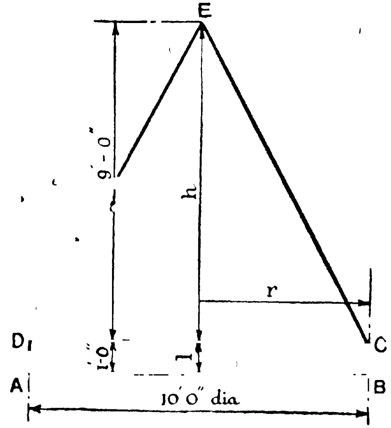

Fig. 39

To this would have to be added all necessary allowances for seams, &c.

FRUSTUMS OF PYRAMIDS AND CONES.—The term frustum is derived from a Latin word meaning piece or bit. It is used mathematically to indicate that piece, bit, or slice of a pyramid or cone which is contained between the base and any plane parallel to the base. The letters ABDE ...... P, fig. 40, represent isometrically the frustum of a regular hexagonal pyramid, and QTVR that of a right circular

cone. ABDEFG and HKLMNP are termed the *ends* of the frustum, and the planes QR and VT are the *ends* of the cone. It should be noted that the two ends in each case are similar figures; that is to say, they are identical in shape though not in dimensions. Thus, HKLMNP is a regular hexagon, similar to,

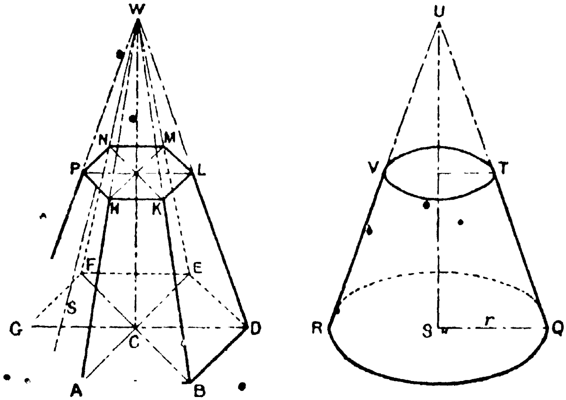

Fig. 40

but less than, ABDEFG; and TV is a circle less than the circle QR.

The slant surface of any pyramidal frustum is made up of a number of trapeziums, one for each side of the base. If the pyramid is right and on a regular base, these trapeziums are equal to each other in all respects. The whole surface of the frustum combines the slant surface and the two ends.

SLANT SURFACE OF FRUSTUM OF RIGHT REGULAR PYRAMID.

Let P be the perimeter of lower end,
p be the perimeter of upper end,

and T be the slant thickness, i.e. the thickness measured along the centre-line of one of the outer lateral faces, as indicated on the face AHPG.

Then, if S be the area of slant surfaces,

$$S = \frac{P + p}{2} \times T;$$

that is to say, the area of the slant surfaces of the frustum of a right regular pyramid is equal to the product of half the sum of the perimeters of the ends and the slant thickness.

VOLUME OF FRUSTUM OF RIGHT REGULAR PYRAMID.—

Let A = the area of the larger end,
a = the area of the smaller end,
t = the vertical height or thickness,
and V = the volume.

$$\text{Then } V = \frac{t}{3}(A + \sqrt{Aa} + a).$$

If the total vertical height h be known, then the volume of the frustum will be,

$$\text{Volume of whole pyramid} = \frac{Ah}{3}.$$

$$\text{Volume of cut-off portion} = \frac{a(h - t)}{3}.$$

$$\text{Volume of frustum} = \frac{Ah}{3} - \frac{a(h - t)}{3}$$

$$= \frac{Ah - a(h - t)}{3}.$$

AREA OF CURVED SURFACE OF FRUSTUM OF RIGHT CONE.—If one keeps in mind the similarity between a regular polygon and a circle, as well as that between a prism and a cylinder, it will be seen that the two perimeters in the frustum of a polygonal figure correspond to the two circumferences of the frustum of a cone; consequently, the $P + p$ in the formula for the former, i.e. in

$$S = \frac{P + p}{2} \times T,$$

are replaced by the signs for the two circumferences, say C for the larger, and c for the smaller; then, the surface area of the frustum of a cone is

$$S = \frac{C + c}{2} \times T.$$

In every case, however, the circumference is equal to π times the diameter, or 2π times the radius. Hence,

if R = the radius of the larger end,
and r = the radius of the smaller end,

$$S = \frac{2\pi R + 2\pi r}{2} \times T$$
$$= T(\pi R + \pi r)$$
$$= \pi T(R + r).$$

VOLUME OF FRUSTUM OF RIGHT CONE.—Again, bearing in mind the similarity mentioned above, and that the volume of a right regular pyramid is represented by

$$V = \frac{t}{3}(A + \sqrt{Aa} + a),$$

the required formula for the frustum of a right cone, with two circles of radii R and r, will be

$$\begin{aligned} V &= \frac{t}{3}(\pi R^2 + \sqrt{\pi R^2 \times \pi r^2} + \pi r^2) \\ &= \frac{t}{3}(\pi R^2 + \pi\sqrt{R^2 r^2} + \pi r^2) \\ &= \frac{\pi t}{3}(R^2 + \sqrt{R^2 r^2} + r^2) \\ &= \frac{\pi t}{3}(R^2 + Rr + r^2). \end{aligned}$$

Although it is unusual to conduct any polygonal calculations in connection with cops, the mule cop is an excellent subject for practice in these calculations.

Example 61.—The dimensions of a cop of cotton

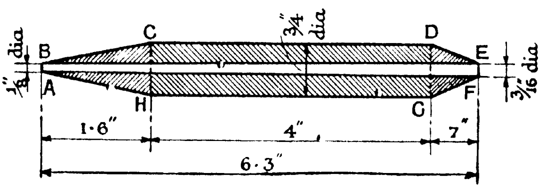

Fig 41

weft appear in fig. 41. An inspection of the dimensioned sketch shows that the complete cop is made up of the frustums of two cones, ABCH and DEFG, together with a cylindrical part CDGH; moreover, the hollow centre of the cop is in the shape of a third frustum of a cone.

$$\begin{aligned} \text{Vol. of cop} &= \text{vol. of ABCH} + \text{vol. of DEFG} \\ &\quad + \text{vol. of CDGH} - \text{vol. of ABEF}. \end{aligned}$$

The right-hand side of the equation shall be taken in stages (1), (2), (3), and (4).

(1) Vol. of ABCH $= \frac{\pi t}{3}(R^2 + Rr + r^2)$ – the hollow part to be taken off later.

$$\therefore V_1 = \frac{\pi \times 1 \cdot 6}{3}[(\tfrac{3}{8})^2 + \tfrac{3}{8} \times \tfrac{1}{16} + (\tfrac{1}{16})^2]$$
$$= \frac{3 \cdot 1416 \times 1 \cdot 6}{3}(\cdot 375^2 + \cdot 375 \times \cdot 0625 + \cdot 0625^2)$$
$$= 1 \cdot 6755(\cdot 1406 + \cdot 0234 + \cdot 0039)$$
$$= 1 \cdot 6755 \times \cdot 1679$$
$$= \cdot 2813 \text{ cu. in.}$$

(2) Vol. of DEFG $= \frac{\pi t}{3}(R^2 + Rr + r^2)$ – the hollow part to be taken off later.

$$\therefore V_2 = \frac{\pi \times 0 \cdot 7}{3}[(\tfrac{3}{8})^2 + \tfrac{3}{8} \times \tfrac{3}{32} + (\tfrac{3}{32})^2]$$
$$= \frac{3 \cdot 1416 \times 0 \cdot 7}{3}(\cdot 375^2 + \cdot 375 \times \cdot 09375 + \cdot 09375^2)$$
$$= \cdot 733(\cdot 1406 + \cdot 0352 + \cdot 0088)$$
$$= \cdot 733 \times \cdot 1846$$
$$= \cdot 1353 \text{ cu. in.}$$

(3) Vol. of CDGH $= \frac{\pi}{4}D^2 l$ – the hollow part to be taken off later.

$$\therefore V_3 = \cdot 7854 \times (\tfrac{3}{4})^2 \times 4$$
$$= 1 \cdot 767 \text{ cu. in.}$$

(4) Vol. of ABEF $= \frac{\pi t}{3}(R^2 + Rr + r^2)$.

$$\therefore V_4 = \frac{\pi \times 6 \cdot 3}{3}[(\tfrac{3}{32})^2 + \tfrac{3}{32} \times \tfrac{1}{16} + (\tfrac{1}{16})^2]$$
$$= \frac{3 \cdot 1416 \times 6 \cdot 3}{3}(\cdot 09375^2 + \cdot 09375 \times \cdot 0625 + \cdot 0625^2)$$
$$= 6 \cdot 5974(\cdot 0088 + \cdot 0059 + \cdot 0039)$$
$$= 6 \cdot 5974 \times \cdot 0186$$
$$= \cdot 1227 \text{ cu. in.}$$

$$\begin{aligned} \text{Vol. } V &= V_1 + V_2 + V_3 - V_4 \\ &= \cdot 2813 + \cdot 1353 + 1 \cdot 767 - \cdot 1227 \\ &= 2 \cdot 1836 - \cdot 1227 \\ &= 2 \cdot 0609 \text{ cu. in. in cop.} \end{aligned}$$

THE SPHERE.—The sphere is that geometrical solid swept out in space by one complete revolution of a semicircle, the diameter of which remains in the same plane and acts as a fulcrum.

Fig. 42 represents such a solid swept out by the semicircle ABD rotating on its diameter AD. The

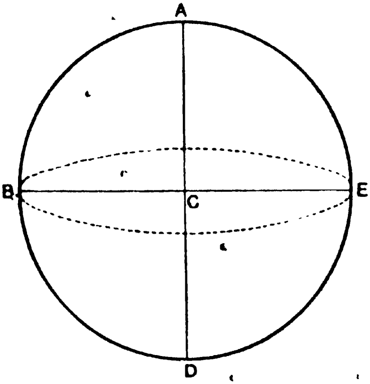

Fig. 42

central point of the diameter AD is the centre C of the sphere; all straight lines drawn from this centre to the outside surface are radii of the sphere and are equal to each other.

The section taken through any plane of the sphere is a circle; if this plane passes through the centre C, the section is termed a central section, or sometimes the great circle of the sphere.

SURFACE AREA OF SPHERE.—It may be proved that the area of the outer surface of a sphere is four times that of its central section.

If A = the surface area,
r = the radius,
and D = the diameter,

the relation may be stated symbolically as under:

In terms of radius.	In terms of diameter.
$A = 4 \times \pi r^2$	$A = 4 \times \frac{\pi}{4} D^2$
$= 4\pi r^2$	$= \pi D^2$
$= 12 \cdot 57 r^2.$	$= 3 \cdot 1416 D^2.$

VOLUME OF SPHERE.—The volume V of a sphere is represented by the following equation:

$$V = \tfrac{4}{3}\pi r^3$$
$$= \frac{4 \times 3 \cdot 1416}{3} r^3$$
$$= 4 \cdot 188 r^3.$$

If required, the volume may also be expressed in terms of the diameter; it is more usual to measure the diameters of spheres than the radii, although, of course, the above equation can always be used by halving the measured diameter.

If D is the diameter, the radius is $\frac{D}{2}$. Substituting $\frac{D}{2}$ for r in the original expression, we have

$$V = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3$$
$$= \frac{4}{3} \times \frac{\pi}{1} \times \frac{D^3}{2^3}$$
$$= \frac{4}{3} \times \frac{\pi}{1} \times \frac{D^3}{8}$$
$$= \frac{\pi D^3}{6} = \frac{3 \cdot 1416 D^3}{6} = \cdot 5236 D^3.$$

SEGMENT OF A SPHERE.—The segment of a sphere is any part cut off the sphere by means of a plane. Fig. 43 shows the part ADB of the sphere cut off by a plane passing through AB. AEB is also a segment of the sphere. If the plane passes through the centre of the sphere, it will divide the sphere into two

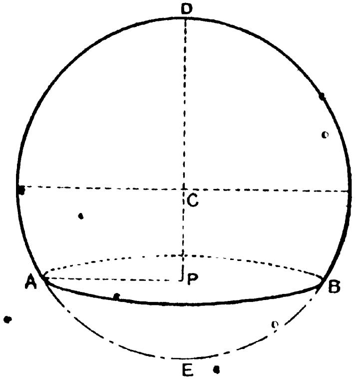

Fig. 43

equal segments called hemispheres—literally, half-spheres.

The base of any spherical segment is a circle. The height of the segment is given by a line drawn at right angles to the base centre, as PD, fig. 43.

CURVED SURFACE OF SEGMENT.—It may be proved that,

if R = the radius of the complete sphere,
h = the height,
and S = the curved surface of the segment,

$$S = 2\pi Rh$$
$$= 6.2832Rh.$$

If desired, the equation may be stated in terms of the diameter D of the complete sphere, thus,

$$\begin{aligned} S &= 2\pi\frac{D}{2}h \\ &= \pi Dh \\ &= 3\cdot1416Dh. \end{aligned}$$

VOLUME OF SEGMENT.—The volume of a segment may be found as under, where

V = the volume of the segment,
r = the radius,
d = diameter,
h = the height from the base,

$$\begin{aligned} V &= \pi h^2\left(r - \frac{h}{3}\right) \\ &= \pi h^2\left(\frac{3d}{6} - \frac{2h}{6}\right) \\ &= \frac{\pi h^2}{6}(3d - 2h). \end{aligned}$$

ZONE OF A SPHERE.—The zone of a sphere is that part of the sphere which is cut off by, or lies between, any two parallel planes. Thus, ABCD, in fig. 44, represents such a solid, its thickness being EF, the line drawn between the surfaces of the two circular ends, and perpendicular to them.

CURVED SURFACE OF ZONE.—The curved surface of a zone may be found from the following data:

R = the radius of the complete sphere,
t = the thickness of the zone,
and S = the curved surface of the zone,

the equation being,

$$\begin{aligned} S &= 2\pi Rt, \text{ or } S = \pi Dt \\ &= 6\cdot2832Rt = 3\cdot1416Dt. \end{aligned}$$

Notice that in both cases the curved surface is equal to

(circumference of sphere) × (thickness of zone),

$$2\pi r \times t,$$

$$\pi D \times t.$$

In other words, the area of the curved surface of the zone of a sphere depends only on the radius of the

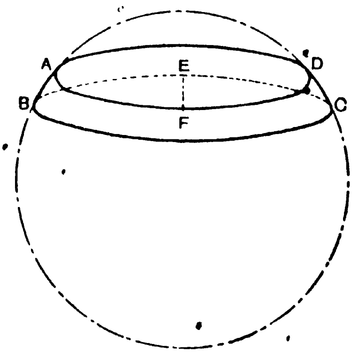

Fig 44

sphere and the thickness of the zone. It follows that all the zones of a sphere, provided they are equal in thickness, have equal curved surface areas.

VOLUME OF ZONE.—With the following data,

V = the volume of the zone,
r_1 = the radius of one circular end,
r_2 = the radius of the other circular end,

the equation for the volume of a zone is represented by

$$V = \frac{\pi t}{6}\{3(r_1^2 + r_2^2) + t^2\}.$$

It may be an advantage to point out again that the subscript numbers 1 and 2 in r_1 and r_2 are used only for the sake of distinguishing between the different radii. The term r_1^2 is read, r one squared, and the term r_2^2 is read, r two squared, the upper figure in both cases representing the index of the power to which r_1 or r_2 is to be raised. Subscript numbers, the lower ones in the above terms, should only be used when it is desirable to keep to a particular letter symbol, and thus avoid the introduction of more than one letter symbol for the same kind of quantity, or for a term which may have many different values.

Example 62.—It is calculated that the weight of each of the governor balls of a certain engine should be 60 lb. If they are to be made of cast iron weighing 0·263 lb. per cubic inch, find the common diameter of the balls.

$$\text{Volume of ball} = \frac{\text{weight of ball in pounds}}{\text{weight of 1 cu. in. of cast iron}}$$

$$= \frac{60 \text{ lb.}}{\cdot 263} = 228 \cdot 14 \text{ cu. in.}$$

$$\text{Volume of sphere} = \cdot 5236 D^3.$$

$$\therefore D^3 = \frac{\text{Volume of sphere}}{\cdot 5236}$$

$$D = \sqrt[3]{\frac{V}{\cdot 5236}}$$

$$= \sqrt[3]{\frac{228 \cdot 14}{\cdot 5236}}$$

$$= \sqrt[3]{435 \cdot 714}.$$

$$= 7 \cdot 58 \text{ in.}$$

See Chapter VI, on Logarithms, in Part II, for method of finding the value of $\sqrt[3]{435 \cdot 714}$.

Example 63.—Fig. 45 is an outline drawing of the interior of a vertical boiling-keir. Find the cubical

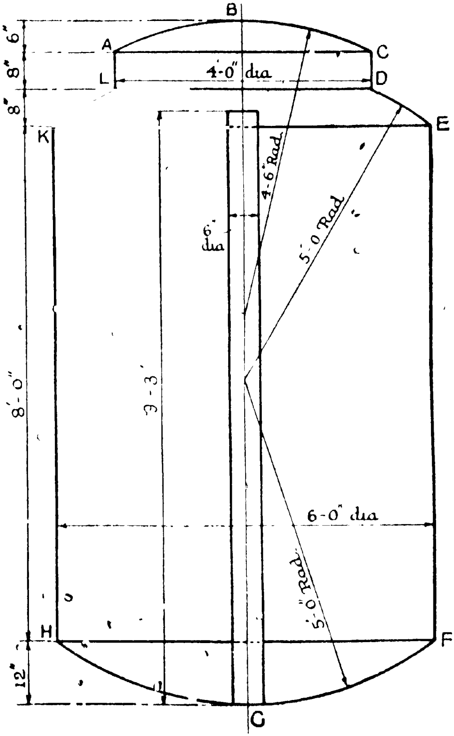

Fig. 45

contents of the large spherical segment FGH, and the spherical zone DEKL.

Volume required = vol. of segment FGH + vol. of zone DEKL.

$$
\begin{aligned}
V_r &= V_s + V_z \\
&= \frac{\pi h^2}{6}(3d - 2h) + \frac{\pi t}{6}[3(r_1^2 + r_2^2) + t^2] \\
&= \frac{\pi h^2}{3}(3r - h) + \frac{\pi \times \frac{2}{3}}{6}[3(3^2 + 2^2) + (\tfrac{2}{3})^2] \\
&= \frac{3{\cdot}1416 \times 1^2}{3}(3 \times 5 - 1) + {\cdot}3491[3(9 + 4) + \tfrac{4}{9}] \\
&= \frac{3{\cdot}1416 \times 14}{3} + {\cdot}3491 \times 39\tfrac{4}{9} \\
&= 14{\cdot}66 \text{ cu. ft.} + 13{\cdot}77 \text{ cu. ft.} \\
&= 28{\cdot}43 \text{ cu. ft.}
\end{aligned}
$$

NOTE.—All the above dimensions are taken in feet; the result is thus in cubic feet.

Exercises, with answers, on p. 121.

EXERCISES

Chapter II, pp. 2 7

1. Write down the values of the following:

(1)	$3a - a$.	*Ans.*	$2a$.
(2)	$x - x$.	,,	0.
(3)	$a \times b$.	,,	ab.
(4)	$x \times 2$.	,,	$2x$.
(5)	$3b + b$.	,,	$4b$.
(6)	$2c + 3c$.	,,	$5c$.
(7)	$2 \div x$.	,,	$\frac{2}{x}$.
(8)	$x \div 3$.	,,	$\frac{x}{3}$.

2. Find the value of $3x$ when:

(1)	$x = 2$.	*Ans.*	6.
(2)	$x = \frac{1}{2}$.	,,	$1\frac{1}{2}$.
(3)	$x = -\frac{1}{2}$.	,,	$-1\frac{1}{2}$.
(4)	$x = 4a$.	,,	$12a$.
(5)	$x = -6a$.	,,	$-18a$
(6)	$x = -3$.	,,	-9.

3. Put into words the following algebraical statement:

$$x = \frac{a \times b \times c \times d}{e + f + g} - \frac{h}{k}.$$

4. Express in symbols:—The area of a circle equals ·7854 times the square of the diameter.

Ans. Let A = area and d = diameter, then $A = \cdot 7854d^2$.

5. The width of a hexagon nut across the flats is given by the formula: $W = 1\frac{1}{2}d + \frac{1}{8}$ in., where W = the width, and d = the diameter of the bolt. Find W when d is $\frac{5}{8}$ in. *Ans.* $W = 1\frac{1}{16}$ in.

6. Find the value of $\frac{x}{3}$ when:

(1) $x = 4{\cdot}5$.	*Ans.*	$1\frac{1}{2}$.
(2) $x = 18$.	,,	6.
(3) $x = 180$.	,,	60.
(4) $x = {\cdot}0036$.	,,	${\cdot}0012$.
(5) $x = -9$.	,,	-3.
(6) $x = 3a - 6b$.	,,	$a - 2b$.

Chapter III, pp. 7-10

1. What are the values of:

(1) $8 + (6 - 2)$.	*Ans.*	12.
(2) $9 - (3 + 1)$.	,,	5.
(3) $15 - (-6 + 3)$.	,,	18.
(4) $18 - (-2 - 8)$.	,,	28.
(5) $-x - (x + x)$.	,,	$-3x$.
(6) $9xy - (6xy + 3xy)$.	,,	0.

2. Prove, by removing the brackets, that:

(1) $(6a - 18) + (3a + 12) - (4a + 20) = 5a - 26$.
(2) $2(a + b) + (2a - b) - 4a - b = 0$.

3. Prove, by simplification, that:

(1) $\frac{2x - 4}{2} + \frac{36x + 90}{18} - \frac{18x + 36}{3} = -3x - 9$.

(2) $\frac{-x^2}{2} + \frac{4x^2 + 16}{4} - \frac{3x^2}{8} = \frac{x^2}{8} + 4$.

4. Find the value of:

(1) $(3a + b) - (2a - 2b) + (6a + b)$ when $a = 4$; $b = 2$.
(2) $(x - y) + 3x^2 - (x + y)$ when $x = 3$ and $y = -$

Ans. (1) $= 36$. (2) $= 37$.

5. Simplify the following:

(1) $\frac{7 - 9x}{3} + \frac{8 + 4x}{5} - \frac{20 + 2x}{30}$. *Ans.* $\frac{49 - 34x}{15}$

(2) $\frac{14 - 2a}{21} + \frac{7a - 3}{7} - \frac{5a - 5}{42}$. *Ans.* $\frac{32a + 15}{42}$

6. Evaluate:

(1) $\cdot 7854\,(a + b)\,(a - b) \times x \times \cdot 263$ when $a = 10$, $b = 5$, and $x = 9$. *Ans.* 139·43.

(2) $A = t\,(a - b - t)$. Find A when $t = \cdot 5$, $a = 6$, and $b = 4$. *Ans.* ·75.

Chapter IV, pp. 11 16

1. A merchant buys cloth at x shillings per yard, and sells it at y shillings per yard. What is his profit per yard? *Ans.* $(y - x)$ shillings.

2. How many shillings are there in £x? How many £ are there in x shillings?

Ans. $20x$ shillings. £$\frac{x}{20}$.

3. A card cylinder makes 125 revolutions per minute. How many revolutions will it make in m seconds?

Ans $\frac{25m}{12}$ revolutions.

4. A loom runs at 180 effective picks per minute, and is weaving cloth with 30 shots per inch. How many yards of cloth will it weave in h hours? *Ans.* $10h$ yd.

5. The horse power transmitted by a single belt is given by the formula: $H.P. = \frac{45\,WS}{33{,}000}$, where W = the width in inches, and S = the speed of the belt in feet per minute. Find the H.P. when W is 8 in., and the speed is 2200 ft. per minute. *Ans.* 24 H.P.

6. Express the following statement as a mathematical formula: the nominal horse-power of a Lancashire boiler supplying steam to a modern engine equals the product of 8, and the area of the fire-grate in square feet.

Ans. $H.P. = 8F.$

where H P. = horse-power, and F = fire-grate area in square feet.

7. The overall length of a spinning-frame of a certain type equals the product of the pitch of the spindles and the number of spindles per side, plus 3 ft. $2\frac{1}{2}$ in. Express this

as a formula, and use the formula to find the overall length of a frame of 80 spindles, 4-in. pitch.

Ans. $L = pN + 3$ ft. $2\frac{1}{2}$ in.
29 ft. $10\frac{1}{2}$ in.

8. Find the weight of a cotton driving-rope, $1\frac{1}{2}$-in. diameter, and 80 ft. long, if w, the weight in pounds per foot, is equal to $\cdot 27d^2$, d being the diameter.

Ans. 48·6 lb.

9. The working stress in pounds of a cotton driving-rope equals $160d^2$, where d is the diameter of the rope in inches. Find the working stress in a $1\frac{3}{4}$-in. diameter cotton rope. *Ans.* 490 lb.

10. The twist or turns per inch of a certain type of flax yarn is equal to twice the sq. root of the count of the yarn where the count is equal to the number of leas of 300 yd. each per pound. Express this symbolically, and use the result to obtain the twist for 40, 80, and 120 lea yarn.

Ans. $T = 2\sqrt{C}$, where T = twist, C = count.
12·65. 17·89. 21·91.

11. The speed of a certain class of looms (up to approximately 100-in. reed space) in picks per minute is equal to the difference between a constant number 196 and the reed space of the loom. Express this as an equation, and find therefrom the speed of a 46-in. reed-space loom, and a 92-in. reed-space loom. P = picks. C = constant. R = reed space.

Ans. $P = C - R$.
i.e. $P = 196 - R$.
150 picks per minute. 104 picks per minute.

12. If the actual speeds of certain types of looms vary inversely as the sq. root of the reed space, what should be the speed of a 92-in. reed-space loom if 150 picks per minute is a suitable speed for a 46-in. reed-space loom?

Ans. 106 picks per minute.

Chapter V, pp. 16-23

1. The overall floor-space for a heavy jute card is 9 ft. $10\frac{3}{4}$ in. by 15 ft. 3 in. Find the area occupied in square feet. *Ans.* 150·94 sq. ft.

2. The overall floor-space for a 280 spindle, $2\frac{1}{2}$-in. gauge ring spinning-frame (140 spindles on each side) is 31 ft. 8 in. by 3 ft. If 40 machines are to be arranged in 2 rows of 20 each, with 5 ft. between each pair of machines, 4 in. between end of pulley-shaft and wall to enable pulley to be removed, 6 ft. for the central pass, and 8 ft. at each end of the flat, find the floor-space required.

Ans. 11,970 sq. ft.

3. A piece of cloth is 40 in. wide and 100 yd. long. Find its value if a square yard costs 3s. *Ans.* £16, 13s. 4*d.*

4. How many yards of 27-inch carpeting would be required to cover the floor of a rectangular room 12 ft. by 16 ft., no allowance being made for fitting, &c.

Ans. 28·44 yd.

5. In a jute breaker card, 6 ft. wide, 40 lb. of material are spread over a length of 13 yd. Find the weight of material spread on the feed-cloth in pounds per square yard. *Ans.* 1·538 lb. per square yard.

6. A small sample of bleached union huck (huckaback) measures $2\frac{1}{4}$ in. by $1\frac{3}{4}$ in., and weighs 11·7 gr. If the cloth is 25 in. in width, find its weight in ounces per yard.

Ans. 5 oz. per yard.

7. Two drop-curtains for a small stage are to be made from cotton cloth; the width of the stage (as well as the finished width of the 2 curtains) is 20 ft., and the height of the upper surface of the curtain support from the floor is 10 ft. If 6 in. extra are required at the top for a draw-slot, and 3 in. at the bottom and at each side of the 2 curtains for hems, find the total area of the cloth required in square yards. *Ans.* [illegible] sq. yd.

Chapter VI, pp. 23 30.

1. A yarn chute is to be erected between two mill buildings 40 ft. apart, from a point 30 ft. from the ground on one wall to a point 10 ft. above the ground on the other wall. Find the length of the chute outside the walls.

Ans. 44·72 ft.

2. The fallers in a flax spread-board lie at an angle of 14 deg. 20 min. to the horizontal. At what angle do they lie to the vertical? *Ans.* 75 deg. 40 min.

3. What would be the area of a triangular scraper of $2\frac{1}{2}$-in. side, and how many could be cut from a piece of sheet steel $25\frac{1}{4}$ in. long by 15 in. broad, neglecting any waste due to cutting?

Ans. 2·706 sq. in. 140 complete scrapers.

4. The end wall of the upper flat of a factory is 30 ft. wide, 10 ft. high to the rafters, and 18 ft. from the floor to the ridge of the roof, all inside measurements. Find the cost of painting two such end walls at 2*d.* per square yard.

Ans. 15*s.* $6\frac{2}{3}$*d.*

5. The area of a triangular plot of ground is 600 sq. yd. Find the length of the base if the perpendicular between the base and the vertex is 60 ft. *Ans.* 60 yd.

6. A firm of bleachers is offered the use of a triangular field, the sides of which measure 18, 27, and 29 chains respectively, the rent being £100. The firm has already been offered equally suitable ground at £4 per acre. Which offer should be accepted?

Ans. The first, since it is fully 4*s.* $3\frac{1}{4}$*d.* per acre cheaper.

7. Make use of the information given in *Example 19*, p. 30, to deduce a special formula for finding the area of an equilateral triangle, given the length of one side. Use this formula to check the result found in Exercise 3 above.

Ans. If A = area, and a = side, then $A = \sqrt{3}\dfrac{a^2}{4}$.

8. A rope pulley rim is to be built in four sections and bolted to a wheel 14 ft. 2 in. in diameter. The four joints are to be machined, and, in order to mark off the limits of the machining, it is necessary to know the length of the chord. (See definition of chord in Chap. X). If the unfinished bore of the rim is 14 ft. $1\frac{1}{2}$ in. in diameter, find the length of the chord.

Ans. Chord $= \sqrt{2 \times 84{\cdot}75\ \text{in.}^2} = 9$ ft. 11·9 in.

9. An isosceles triangle has two sides, each $4\frac{1}{2}$ ft., and a base of 7·484 ft. Find the altitude. *Ans.* 2·5 ft.

10. Find the area of a triangle, the base of which is 7·484 ft., and the vertical height 2·5 ft.

Ans. 9·355 sq. ft.

Chapter VII, pp. 30–37

1. The diagonals of a rhombus intersect at right angles, hence, obtain a formula for the area of a rhombus in terms of the diagonals.

NOTE.—Area of triangle $= \dfrac{\text{base} \times \text{altitude}}{2}$.

Ans. Let A = area, d_1 and d_2 = the diagonals, then $A = \dfrac{d_1 d_2}{2}$.

2. The area of a parallelogram is 90 sq. yd., and its base measures 30 ft. Find the perpendicular distance between two parallel sides. *Ans.* 27 ft.

3. A rhombus of 16-inch side has an altitude of 15 in. Find its area in square feet. *Ans.* $1\frac{2}{3}$ sq. ft.

4. A size- or starch-trough for a dressing-machine is shaped in section like a trapezium, 12 in. wide at the bottom, and 18 in. wide at the top. If the full depth of the trough is 9 in., find the area of its cross section (equivalent to the end) in square inches.

Ans. 135 sq. in.

5. Find the rent, at £5 per acre, of a mill site in the form of a trapezium, one diagonal of which measures 18 chains, and the off-sets from this diagonal to two opposite vertices being 10 chains and 9 chains respectively.

Ans. £256, 10*s.* 0*d.*

6. A trapezium ABCD has a diagonal AC 60 ft. long, while sides AB and BC are respectively 50 ft. and 40 ft. in length. If the off-set from point D to AC is 35 ft. long, find the area in square yards. *Ans.* 226·9 sq. yd.

Chapter VIII, pp. 37–41

1. A hexagonal bar of brass, used in making nuts for the starch-boxes of dressing-machines, measures $\frac{7}{8}$ in. across one face, and 8 ft. in length. What is the weight in pounds if 1 cu. ft. of brass weighs 518 lb?

Ans. 57·24 lb.

2. A regular hexagon is composed of 6 equal equilateral triangles; deduce a special formula for finding the area. (Refer, if necessary, to p. 28, Chap. VI).

Ans. $2 \cdot 598b^2$.

3. Use the formula $2 \cdot 598b^2$ to find the area in square feet of a regular hexagon of 18 in. side.

Ans. 5·846 sq. ft.

4. If, in a dust-extraction plant, 1 sq. in. of pipe area is allowed for each 20 cu. ft. of air passing per minute, what volume of air in cubic feet per hour will pass through a hexagonal pipe of 6-in. side?

Ans. 112,233·6 cu. ft. per hour.

5. On a supposition similar to that in Question 4 above, find the length of side of a hexagonal pipe which will pass 100,000 cu. ft. of air per hour. *Ans.* 5·663 in.

6. Construct to any suitable scale a regular hexagon of 6-in. side, and letter the vertices A, B, C, D, E, F, and the centre O. Draw the 3 diagonals AD, BE, and CF, dividing it into 6 equal equilateral triangles. Find the area of the hexagon, and prove that it is equal to twice the area of the triangle obtained by joining AC, CE, and EA.

7. An octagonal dyeing-reel is made of pitch pine; the width of each face is 2 in., the width across the flats is 4·828 in., and the length is 4 ft. Find its weight in pounds, if 1 cu. foot of pine weighs 34½ lb. *Ans.* 18·37 lb.

8. A 12-yd. circumference vertical warping-mill has 24 spokes. Find the floor-space occupied in square feet, it being given that the perpendicular drawn from the centre to the centre of one of the sides is 5 ft. 8⅜ in.

Ans. 102·56 sq. ft.

Chapter IX, pp. 41–56

1. The centre post of a vertical warping-mill is 2¼ in. diameter, the cord which is wrapped round the post being ⅛ in. diameter. What length of cord will the centre post give off or take on in 12 revolutions of the mill? Use $\pi = 3 \cdot 14$. *Ans.* 89·49 in.

2. A main driving-belt pulley is 8 ft. in diameter, and runs at 100 revolutions per minute. Find the speed of the belt in feet per minute. Use $\pi = 3 \cdot 14$. If another

pulley on the same shaft is to have a belt speed of 2000 ft. per minute, what should be the diameter of the pulley?

Ans. 2512 ft. per minute. 76·43 in. diameter.

3. A roving-frame delivers l inches of sliver per minute per spindle; the sliver is twisted by the flyer and coiled regularly upon a rove-bobbin. Find the effective speed of the bobbin when the diameter over the yarn is x inches. Then use the expression obtained to find the actual effective speeds of the bobbin when l is 800, and x is 2 in., 3 in., and 4 in. diameter.

Ans. $\frac{l}{\pi x}$ r.p.m. 127·4, 84·9, 63·7 r.p.m.

4. A beam of cotton yarn weighs 150 lb. net. The diameter of the empty beam is 5 in., and the diameter over the beam plus yarn is 14 in. What weight of yarn would be on the beam when the diameter over the partially-filled beam is 10 in.

Ans. 65·8 lb. with logs.
65·79 lb. ordinary method.

5. A warp-beam is 6 in. diameter and has 24 in. diameter flanges. It is intended to put a chain of 10 cuts (pieces) upon it in a dry beaming-machine. After the first cut has been beamed, it is found that the diameter over the yarn is 9½ in. Will the warp-beam hold the 10 cuts?

Ans. No, unless "piling" is resorted to. The sectional area = 540 sq. in., 1 cut occupies 54·25 sq. in. sectional, hence 10 cuts would occupy 542·5 sq. in. sectional. Room, without piling, for 9·954 cuts.

6. The yarn on a warp-winding bobbin (see Exercise 2, Chap. XIII) weighs 1½ lb. and contains 24 leas of 300 yd. each of 16ᶠ linen yarn; the volume of yarn on the bobbin is 67·75 cu. in. (See Exercise 1, Chap. XIII). Find the *approximate* diameter of the yarn.

Ans. ·01824 in. or $\frac{1}{55}$ in.

7. The diameter d of a linen yarn is equal to $\frac{1}{k\sqrt{c}}$, where k is a constant under given conditions, and c is the count of the yarn in leas of 300 yd. each per pound. Find k when c is 16 leas per pound, and d is ·01824. (See Exercise 6 above, and Exercises 1 and 2, Chap. XIII.)

Ans $k = 13·71$.

8. If the actual value of k in Exercise 7 is 16, find the real diameter of 16' linen yarn. *Ans.* $\frac{1}{64}$ in.

9. Deduce a rule to find the area of a circle in terms of its circumference c. Then use it to find the cross-sectional area of a 3-in. rope, noting that rope-makers measure ropes by the circumferences of the latter.

$$Ans.\ \frac{c^2}{4\pi} = \cdot 07958c^2 = \cdot 08c^2. \qquad \cdot 72 \text{ sq. in.}$$

Chapter X, pp. 56 64

1. A top roller for a leaf in a loom moves through an angle of 270 deg., and with the thickness of the strap has an effective diameter of 2 in. Find the length of the strap given off when the leaf descends, and length taken on when the leaf rises, i.e., find the travel of the leaf.
Ans. 4·7124 in.

2. A top roller is attached by a strap to a leaf which has a 4-in. travel. Find the effective diameter of the top roller if the arc of movement must not exceed 285 deg.
Ans. 1·608 in. diameter.

3. The breast beam of a certain loom is 48 in. wide and the front is convex in shape; if the width at the ends of the breast beam is 5 in., and at the centre $5\frac{3}{8}$ in., find the radius of curvature.
Ans. 64·0156 ft. or 64 ft. $0\frac{3}{16}$ in.

4. Refer to Fig. 45 and to Example 63, and find the complete volume of the vertical kier, remembering that the volume occupied by the puffer-pipe or vomiting-pipe in the centre must be deducted.

Ans. Segment	FGH	14·661	cu. ft.
Zone	DEKL	13·769	,,
Segment	ABC	3·404	,,
Cylinder	AGDL	8·377	,,
Cylinder	EFHK	226·195	,,
		266·406	,,
Puffer-pipe		1·816	,,
Total		264·590	cu. ft.

5. If the cloth in the vertical kier, fig. 45, occupies the cylindrical portion only, i.e. EFHK, find what fraction of the total cubical contents is represented by the above cylindrical portion.

Ans. $\frac{224 \cdot 024}{264 \cdot 590} = \cdot 8489$.

6. The section of a Lancashire boiler is 9 ft. diameter inside; if the level of the water is 2 ft. from the top, find the width across the surface of the water, i.e. the chord. Let $c = \frac{1}{2}$ chord; h = height from water level to top; r = radius. (Consult Example 40). *Ans.* 7·484 ft.

7. The level or surface of water in a 9 ft. Lancashire boiler is 2 ft. from the top. This level of water represents a chord equal to 7·484 ft. (see answer to Question 6 above.) If the two radii drawn to the two ends of the chord make an angle of 180 − (2 × 33 deg. 45 min.), find the arc of the segment. *Ans.* 8·833 ft.

8. A sector in a 9-ft. diameter circle subtends an angle of $112\frac{1}{2}$ deg. Find the area. *Ans.* 19·87 sq. ft.

9. In connection with Exercise 8, find the difference in area between the sector and that of the hemisphere.

Ans.	Hemisphere	31·80 sq. ft.
	Sector	19·87 sq. ft.
	Difference	11·93 sq. ft.

10. The area of a sector of a circle is 19·87 sq. ft., and the area in the triangle below the chord of such sector is 9·355 sq. ft. What is the area of the segment above the triangle? *Ans.* 10·515 sq. ft.

11. A cylindrical boiler of the Lancashire type is 30 ft. in length by 9 ft. in diameter; the two flues which extend from end to end of the boiler are each 3 ft. 5 in. in diameter. If the volume above the level of the water in the boiler is 315·45 cu. ft. (10·515 sq. ft. × 30 ft.), calculate the approximate number of gallons of water in the boiler, and the volume occupied by it.

Ans.

Vol. of boiler	−	vol. of tubes	−	vol. of segment	=	cu. ft.
1008	−	550	−	315·45	=	1042·55.

1042·55 × 6·25 = 6516 gall.

Chapter XI, pp. 65–69

1. A spur-wheel has 120 teeth; if the diametral pitch is 6, what is the diameter of the wheel?

Ans. 20 in.

2. A wheel of 10 in. diameter has 6 teeth in 3·1416 in. of the pitch circle. How many teeth are there in the wheel?

Ans. 60.

3. The diameter of the pitch circle of a wheel with 60 teeth is 10 in. Find the circular pitch and the diametral pitch. *Ans.* ·5236 in.; 6.

4. If cast-iron moulded gear-wheels should not have a rim-speed (taken at the pitch circle) greater than 1000 ft. per minute, what is the largest number of teeth of No. 4 pitch required to run at 200 r.p.m.

Ans. 76·4, i.e. 76 teeth.

5. The crank-shaft pinion of a loom has 26 teeth, No. 3 pitch, and gears with the bottom or low shaft-wheel of 52 teeth. Find the distance between the shafts, centre to centre. *Ans.* 13 in.

6. A spur-wheel of 96 teeth, $1\frac{1}{2}$ in. circular pitch, runs at 100 r.p.m. Find the speed at the pitch circle in feet per minute. *Ans.* 1200 ft. per minute.

7. Deduce a rule for converting diametral (or Manchester) pitch into circular (or true) pitch. Use it to find the equivalent in circular pitch of No. 2 diametral pitch.

Ans. $P_c = \dfrac{\pi}{P_d}$. 1·5708 circular pitch.

8. The overall or outside diameter (at points of teeth) of a spur-wheel $= \dfrac{N+2}{P_d}$, where N is the number of teeth, and P_d is the diametral pitch. Find the outside diameter of a spur-pinion of 18 teeth, $1\frac{1}{4}$ in. circular pitch. *Ans.* 7·958 in.

Chapter XII, pp. 69–75

1. The weights and dimensions of three different kinds of bales containing cotton fibre are:

I = Indian: 396 lb. 48 in. × 20 in. × 18 in.
E = Egyptian: 720 lb. 51 in. × 31 in. × 22 in.
B = Brazilian: 220 lb. 49 in. × 20 in. × 18 in.

From the above particulars, calculate:

1. The volume in cubic feet occupied by each.
Ans. I = 10. E = 20·13. B = 10·21.

2. Their relative densities in pounds per cubic foot.
Ans. I = 39·6. E = 35·77. B = 21·55.

3. The specific gravity of the cotton in the baled state. Note. Use 996·5 oz. per cubic foot of water.
Ans. I = ·6357. E = ·5744. B = ·3459.

4. The weight of each kind carried per ton rate, 1 shipping ton reckoned equal to 40 cu. ft.
Ans. I = 14·143 cwt. E = 12·775 cwt. B = 7·696 cwt.

2. A ring-spinning-frame packed for shipment weighs 147 cwt. and measures 500 cu. ft. If 40 cu. ft. = 1 shipping ton, will the freight be charged on deadweight or measurement? *Ans.* 147 cwt. = 7 tons, 7 cwt. by weight.

$$\frac{500 \text{ cu. ft.}}{40} = 12\tfrac{1}{2} \text{ tons, by measurement.}$$

Freight would be charged by the latter.

3. The inside measurements of a rectangular tank are: 15 ft. long, 10 ft. broad, and 5 ft. deep. How many gallons of water will it hold, if 6¼ gall. occupy 1 cu. ft.
Ans. 4687½ gall.

4. The tank mentioned in Exercise 3 is to be sunk level with the ground. How many cubic yards must be excavated if the sides, ends, and bottom are 3 in. thick? What weight of earth in tons will be removed, if its density is 90 lb. per cubic foot?
Ans. 31·65 cu. yd. 34·33 tons.

5. A farina size or starch mixture equal to 236 gall. is made from 126 lb. of farina, 7 lb. of gloy, and 200 gall. of water. What is the probable specific gravity of the starch mixture? Take 1 gall. of water = 10 lb. *Ans.* ·9038 s.g.

6. Textile machinery is packed for shipment in 6 cases of the following dimensions:

(1) 4′ 6″ × 3′ 2″ × 2′ 6″
(2) 5′ 5″ × 3′ 10″ × 2′ 4″
(3) 5′ 5″ × 3′ 10″ × 2′ 7″
(4) 6′ 0″ × 2′ 7″ × 2′ 2″
(5) 17′ 1″ × 1′ 5″ × 1′ 11″
(6) 15′ 4″ × 1′ 7″ × 1′ 8″

Find the shipping weight when 40 cu. ft. = 1 shipping ton. *Ans.* 6·45 tons.

Chapter XIII, pp. 76–84

1. A warp-winding bobbin has 4-in. diameter flanges, 1-in. diameter barrel, and is $5\frac{3}{4}$ in. between the flanges. When the bobbin is just full, i.e. the yarn level with the flanges, it contains 24 leas of 16' bleached flax or linen yarn. Calculate the apparent number of cubic inches of yarn on the bobbin. Use $\pi = 3\cdot1416$.

Ans. 67·74 cu. in. 67·75 cu. in. by logs.

2. If the bobbin mentioned in Exercise 1 holds $1\frac{1}{2}$ lb. of yarn, find the specific gravity of the yarn in the wound state, taking 1 cu. ft. of water to weigh 996·5 oz.

Ans. ·6142.

3. Find a formula for determining the whole area of a hollow cylinder. Use it to find the weight of sheet-lead, $\frac{1}{16}$ in. thick, required to cover completely a hollow cylinder open at both ends. The external diameter of the cylinder is 24 in., the thickness is 2 in., and the length is 48 in. The lead weighs ·406 lb. per cubic inch.

Ans. $\pi(D + d)\left(l + \dfrac{D - d}{2}\right)$. 175·4 lb.

4. The worker of a flax card is 5 ft. 11 in. wide between its flanges, and 5 in. in diameter. Find the length of card clothing, or filleting, 2 in. wide required to cover its surface. *Ans.* 46·47 ft. or 46 ft. $5\frac{5}{8}$ in.

5. The pressing-roller of a jute drawing-frame measures 8 in. diameter by 6 in. wide. Neglecting overlap, find the area in square inches of the leather required to cover completely its curved surface. Use $\pi = 3\cdot1416$.

Ans. 150·7968 sq. in.

6. The bore of a drain-pipe from a washing-machine is 3 in. diameter; if the water passes through it at the rate

of 1 ft. per second, how many gallons will it discharge per hour? *Ans.* 1104·48 gall.

Chapter XIV, pp. 84–105

1. The separator of a dust-extraction plant is made in two parts: a cylindrical tube, 10 ft. diameter by 3 ft. deep, and a truncated cone (or frustum) 10 ft. diameter at one end, 18 in. diameter at the other, and vertical height 12 ft. Find its capacity in cubic feet *Ans.* 603·9 cu. ft.

2. Find the number of square feet of galvanized iron in the above separator, making no allowances for waste in cutting, joints, &c. *Ans.* 324·2 sq. ft.

3. On a horizontal line AB as diameter, and to any suitable scale, construct a semicircle of 30 in. radius. At right angles to AB measure 13 in., and through this point draw a chord CD parallel to AB. At a further distance of 11 in. above CD draw a second parallel chord EF, leaving a segment 6 in. high. Let the semicircle represent the elevation of a hemisphere. Measure CD and EF very carefully indeed. Then use the various formulae discussed in Chap. XIV, and prove, by substitution, that the volume of the hemisphere equals the sum of the volumes of the two zones and the segment.

N.B.—A slight difference may be found which will be due to inaccuracies of measurement.

4. An army bell-tent has dimensions as in Fig. 39. Find its capacity, and the number of cubic feet of air-space available for each of the 8 men who occupy it.

Ans. 314·16 cu. ft. 39·27 cu. ft. per person.

5. The following four expressions appear in different places for the value of the volume of a segment. Prove that they are of equal value. r = radius; d = diameter; h = height.

(1) $\pi h^2\left(r - \frac{h}{3}\right)$.

(2) $\cdot 5236h^2(3d - 2h)$.

(3) $\frac{\pi}{3}(3rh^2 - h^3)$.

(4) $\frac{\pi h^2}{3}(3r - h)$.

6. If the formula for the volume of a segment is $\frac{\pi h^2}{3}(3r - h)$, prove that when h is equal to the diameter, or twice the radius, the above formula assumes that for a sphere, i.e. $\frac{4}{3}\pi r^3$.

7. The formula for the volume of a sphere $= \frac{4}{3}\pi r^3$.

The formula for the volume of a hemisphere $= \frac{2}{3}\pi r^3$.

Volume of hemisphere − volume of zone = volume of segment.

If the formula for the volume of a zone be

$$\frac{1}{3}\pi(r - h)[3r^2 - (r - h)^2],$$

prove that:

$$\frac{2}{3}\pi r^3 - \frac{1}{3}\pi(r - h)[3r^2 - (r - h)^2] = \frac{\pi h^2}{3}(3r - h).$$

8. An ordinary galvanized-iron bucket is 9 in. deep, 12 in. diameter at the top, and $7\frac{1}{2}$ in. diameter at the bottom. How many gallons of water will it hold when filled to the top? *Ans.* 2·474 gall.